

Whole-Herd Optimization with the Cornell Net Carbohydrate and Protein System. II. Allocating Homegrown Feeds Across the Herd for Optimum Nutrient Use

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ABSTRACT

The objective of this paper was to develop a linear optimization procedure for allocating homegrown feeds across the herd to optimize use of their nutrients in whole farm nutrient management planning. The first step involved developing an optimal diet for each group by a linear programming (LP) procedure developed for the Cornell Net Carbohydrate and Protein System (CNCPS). Information (feed biological values, nutrient requirements, animal and environmental factors, nutrient excretion, and the optimal diet) for each group was exported to a whole farm worksheet, where another LP procedure was used to optimize the allocation of homegrown crops by satisfying the set requirements of each animal group while optimizing return over feed costs and nutrient excretion constraints.

A sample evaluation shows how this model was used to reduce N, P, and K excretion by changing feeding strategies while maintaining return over feed costs.

(Key words: linear programming, whole farm nutrient management, Cornell Net Carbohydrate and Protein System, optimization)

Abbreviation key: **BV** = biological value, **CNCPS** = Cornell Net Carbohydrate and Protein System, **CS** = corn silage, **GHa** = grass hay, **HMSC** = high moisture shell corn, **IDM** = indigestible DM, **LP** = linear programming, **ME** = metabolizable energy; **MMLa** = alfalfa silage, **MP** = metabolizable protein.

INTRODUCTION

In a previous study (22), we demonstrated that an excess of nutrients (N, P, and K) on dairy farms could lead to water quality problems. It could be beneficial

if a lesser amount of nutrients was imported and a higher proportion of herd nutrient requirements were met with homegrown feeds.

A recent version of the Cornell Net Carbohydrate and Protein System (**CNCPS**) (5) was developed for use in whole farm feed production and inventory planning and for predicting nutrient excretion. This version of the CNCPS is part of the Cornell University Nutrient Management Planning System (1, 11, 19), which attempts to integrate soil fertility, crop management, crop rotation, animal nutrition, economic factors, and engineering considerations into a workable nutrient management planning system. Unique to this version of the model is the prediction of nutrient excretion. The inclusion of equations to predict annual herd nutrient excretion and urine and fecal production has been described by Tylutki and Fox (19). This allows the user to compute annual herd feed requirements and cost and nutrient excretion. This information can be used to plan annual homegrown crops and purchased feed requirements and the impact of various combinations of homegrown and purchased feeds, herd size, and milk production level on annual returns over feed costs and nutrient balances.

The CNCPS can be effective in decreasing ration costs and excess nutrients in the diet (6, 13, 17, 19). However, on an operating dairy farm, there may be 10 or more animal groups. Therefore, to develop a whole farm nutrient plan, the user has to balance the diets for each group and also has to manually evaluate the impact of the combination of diets developed on farm profit, nutrient excretion, and feed inventories until an optimal answer is found. This task is onerous because so many variables and limitations have to be considered concurrently. Therefore, a method is needed that can shorten the time required to find optimal whole-herd feeding strategies, crop production plans, environmental impact, and potential income over feed costs.

Westphal et al. (23) and Henry et al. (7) used linear programming (**LP**) to study the relationship of plant nutrient management strategies to optimal herd size and net farm return. Nicholson et al. (15) and Urbina

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(20) used an LP model to compare nutritional management strategies for dual-purpose herds in Latin America. However, none has used the structure of the CNCPS to evaluate whole farm nutrient management under North America conditions. In a companion paper, Tedeschi et al. (18) described a procedure for optimizing diets within the structure of the CNCPS.

The purpose of this paper was to describe the structure of the linear optimization model that used the structures of the previously described optimizer to allocate the available homegrown feeds across the herd.

MATERIALS AND METHODS

Model Overview

The whole farm optimization model for the CNCPS system can be stated algebraically as:

$$\begin{aligned} \text{Maximize: } & Z = C'X \\ \text{Subject to: } & AX \geq, =, \text{ or } < B \\ & X \geq 0 \end{aligned}$$

Where:

Z = maximum profit ration (\$/d); or minimum N or P mass balance, (tonne/yr)

C' = $n \times 1$ vector of objective function coefficient (e.g., price of milk, feeds, and fertilizer),

A = $m \times n$ matrix of technical coefficients [e.g., feed metabolizable energy (**ME**), metabolizable protein (**MP**), rumen N, rumen peptide, Ca, P, crop yield)

B = $m \times 1$ vector of constraints (e.g., ME, MP requirement, annual milk production, annual crop consumption, total crop acres, annual nutrients excreted), and

X = $n \times 1$ vector of variables (e.g., feed consumption, crop acres, milk production).

Adjustable Variables

In this model, with the use of the Microsoft Excel 2000 Solver add-in (Microsoft Corp., Seattle, WA), an optimal value for a formula in one cell—called the target cell (objective function) in a worksheet—can be found. The Solver add-in worked with a group of cells that were related, either directly or indirectly, to the formula in the target cell. It adjusted the values in the specified changing cells—called the adjustable cells (feeds, milk production level, cropland acres)—to produce the result specified from the target cell formula. Constraints to restrict the values that the Solver add-in used in the model were set, and the constraints

referred to other cells that affected the target cell formula.

Constraints and Equations

The optimization model had the following components: animal feed and nutrient, cropland and yield, nutrient excretion, economic parameters, and N and P mass balance. Each component had constraints and equations (Table 1) as described below.

Animal feed and nutrient. Calculations of feed biological values (**BV**) and animal requirements were similar to those developed by Tedeschi et al. (18), except that N recycling was accounted for by using an iterative procedure, described later on this section. The CNCPS required an input of milk production, which was used to predict DMI, nutrient requirements, and cost of the ration. The least-cost optimization procedure described by Tedeschi et al. (18) identified the least-cost ration at a given milk production level; however, the least-cost ration was not always the most profitable one for the farm. Equations of the CNCPS were modified to search for the optimum ration within a range of milk production levels rather than a set level. For growing animals, a single value of BW gain (target weight gain) was used to calculate the growth requirement. The constraints and equations used in the animal and feed component are summarized in Table 2.

For nonlactating dairy cattle, DMI prediction equations are the same as those in the CNCPS (5). For lactating dairy cattle, the DMI was a linear function of milk production, so those equations were rewritten to calculate DMI for different milk production levels as described by Fox and Tylutki (3).

$$\begin{aligned} \text{DMI}_{j\text{MIN}} &= ((0.0185 \times \text{BW} + 0.305(0.4 + 0.15 \times \text{PQ}) \times \\ &\quad \text{MILK}_{j\text{MIN}}) \times \text{MUD1} \times \text{TEMP1}) \times \text{lag} \\ \text{DMI}_{j\text{MAX}} &= ((0.0185 \times \text{BW} + 0.305(0.4 + 0.15 \times \text{PQ}) \times \\ &\quad \text{MILK}_{j\text{MAX}}) \times \text{MUD1} \times \text{TEMP1}) \times \text{lag} \end{aligned}$$

Where:

MUD1 is a mud adjustment factor,

TEMP is a temperature adjustment factor, and

$\text{lag} = 1 - \exp(- (0.0564 - 0.124 \times 2) \times (\text{week in milk} + 2.36))$; from Roseler et al. (16)

For lactating cows, ME and MP requirements are driven by milk production (5), therefore, ME and MP equations were rewritten as:

$$\text{ME}_{j\text{MIN}} = \text{ME}_m + \text{ME}_p + \text{ME}_g + (\text{MILK}_{j\text{MIN}} \times ((0.3512 + (0.0962 \times \text{MF}))/0.644))$$

Table 1. Abbreviations and constraints used in the whole farm nutrient optimization model.

Lower constraint	Name	Upper constraint	Unit	Description	
BALNMIN \times r BALPMIN \times r	BALK		tonne/yr	Whole farm K balance within an year	
	BALN	BALNMAX \times R	tonne/yr	Whole farm N balance within an year	
	BALP	BALPMAX \times R	tonne/yr	Whole farm P balance within an year	
CA _j MIN \times r	BV			Biological values	
	BW		kg	BW	
	CA _j	CA _j MAX \times R	g/d	Calcium supply from jth diet and lower and upper requirement constraints	
DMI _j MIN \times r	DMI _j	DMI _j MAX \times R	kg/d	DMI from jth diet and lower and upper requirement constraints	
	DMI _j N		kg/d	Nitrogen in DMI of the jth group average animal	
	DMI _j P		kg/d	Phosphorus in DMI of the jth group average animal	
eNDF _j MIN \times r	ENDF _j	eNDF _j MAX \times R	kg/d	Effective fiber supply from the jth diet and lower and upper requirement constraints	
	EXEN _j		g/d	N excretion of the jth group average animal	
	EXEP _j		g/d	P excretion of the jth group average animal	
	FER _i		kg/ha	Fertilizer applied on the ith home grown feed field	
	FER _i \$		US\$/tonne	Price of the ith fertilizer	
	FER _i K		decimal ¹	Percentage of K in the ith fertilizer	
	FER _i N		decimal ¹	Percentage of N in the ith fertilizer	
	FER _i P		decimal ¹	Percentage of P in the ith fertilizer	
	F _i \$		US\$/tonne	Price of the ith feed	
	F _i \$		US\$/tonne	Price difference between purchased and home grown of the ith feed resources	
	F _i CP		decimal	CP % of the ith feed	
	F _{ij} MIN	F _{ij}	F _{ij} MAX	kg/d	The ith feed supply from the jth diet and lower and upper constraints
		F _i K		%	K of the ith feed
F _i P			%	P of the ith feed	
FPA			g/d	Metabolic fecal protein	
G _j				Animal numbers in the jth group	
GROW _j CA			g/d	Ca requirement for growth of the jth group average animal	
GROW _j N			g/d	N requirement for growth of the jth group average animal	
GROW _j P			g/d	P requirement for growth of the jth group average animal	
HEXENMIN \times r HEXEPMIN \times r	HEXEN	HEXENMAX \times R	tonne/yr	Whole herd N excretion	
	HEXEP	HEXEPMAX \times R	tonne/yr	Whole herd P excretion	
	HF\$		US\$/yr	Whole herd feed expense	
	HFBUYK		tonne/yr	K from purchased feeds	
	HFBUYN		tonne/yr	N from purchased feeds	
	HFBUYP		tonne/yr	P from purchased feeds	
	HFER\$		US\$/yr	Expense for the fertilizers	
	HFERK		tonne/yr	K from purchased fertilizers	
	HFERN		tonne/yr	N from purchased fertilizers	
	HFERP		tonne/yr	P from purchased fertilizers	
HF _i BUYMIN \times r	HF _i	HL _i Y _i	tonne/yr	The ith annual herd feed requirement	
	HF _i BUY	HF _i BUYMAX \times R	tonne/yr	Purchased portion of the ith annual herd feed requirement	
	HL		ha	Total farm land hectares	
	HL _i Y _i		tonne/ha	Crop production from land i grown for crop i	
	HL _i Y _i NFIX		tonne/ha	Nitrogen fixation in field i if crop i is legume	
	MEAT\$		US\$/tonne	Meat price	
	HMEAT\$		US\$/yr	Annual meat sale	
	HMEAT		tonne/yr	Meat sold out of farm boundary	
	HMEATK		tonne/yr	K in meat exported from farm	
	HMEATN		tonne/yr	N in meat exported from farm	
	HMEATP		tonne/yr	P in meat exported from farm	
	HMILK\$		US\$/yr	Annual milk sale	
	HMILKK		tonne/yr	K in milk exported from farm	
	HMILKN		tonne/yr	N in milk exported from farm	
	HMILKP		tonne/yr	P in milk exported from farm	

Continued

Table 1. Continued.

Lower constraint	Name	Upper constraint	Unit	Description
HPROFITMIN $\times r$	HPROFIT	HPROFITMAX $\times R$	US\$	Return over feed costs of the farm
	IDM		g/d	Indigestible dry matter
	Lag			Lag adjustment for DMI
L_i MIN $\times r$	L_i	L_i MAX $\times R$	ha	Hectares of land grown for the i th feed
	LOSS $_i$		decimal ¹	Spoilage loss during harvest and storage
	MEATCP		decimal ¹	CP composition in meat
	MEATK		decimal ¹	K composition in meat
	MEATP		decimal ¹	P composition in meat
	ME $_g$		Mcal/d	Metabolizable energy for gain
ME_j MIN $\times r$	ME $_j$	ME_j MAX $\times R$	Mcal/d	Metabolizable energy supply from the j th diet and lower and upper requirement constraints
	ME $_m$		Mcal/d	Metabolizable energy for maintenance
	ME $_p$		Mcal/d	Metabolizable energy for production
	MF		decimal ¹	Milk fat
	MILK\$		US\$/100 Kg	Milk price
	MILKCP		decimal ¹	CP composition of milk
$MILK_j$ MIN $\times r$	$MILK_j$	$MILK_j$ MAX $\times R$	Kg/d	Milk production of the j th group average animal
	MILKK		decimal ¹	K composition of milk
	MILKP		decimal ¹	P composition of milk
	MP $_g$		g/d	Metabolizable protein for gain
MP_j MIN $\times r$	MP $_j$	MP_j MAX $\times R$	g/d	Metabolizable protein supply from the j th diet and lower and upper requirement constraints
	MP $_m$		g/d	Metabolizable protein for maintenance
	MP $_p$		g/d	Metabolizable protein for production
	MUD1			Mud adjustment factor for DMI
MDF_j MIN $\times r$	NDF $_j$	NDF $_j$ MAX $\times R$	kg/d	NDF supply from the j th diet and lower and upper requirement constraints
	NFC		g/d	Ruminal degraded nonfiber carbohydrate
	ORT		decimal ¹	Percentage of orts in DMI
	PEPTIDE		g/d	Peptide is the sum of ruminal degraded true protein
P_j MIN $\times r$	P_j	P_j MAX $\times R$	g/d	P supply from the j th diet and lower and upper requirement constraints
	PP		%	True milk protein % = Milk $_j$ CP% \times 0.93
	PQ		%	Milk fat
	PREG $_j$ CA		g/d	The j th group average animal's Ca requirement for pregnancy
	PREG $_j$ N		g/d	The j th group average animal's N requirement for pregnancy
	PREG $_j$ P		g/d	The j th group average animal's P requirement for pregnancy
	r		decimal ¹	Minimum limit of the nutrients requirement
	R		decimal ¹	Maximum limit of the nutrients requirement
	RESERVE $_j$ N		g/d	The j th group average animal's N requirement for body reserve N mobilization
	RESERVE $_j$ P		g/d	The j th group average animal's P requirement for body reserve P mobilization
$RUMN_j$ MIN $\times r$	RUMN $_j$	RUMN $_j$ MAX $\times R$	g/d	Rumen nitrogen balance from the j th diet and lower and upper requirement constraints
$RUMPEP_j$ MIN $\times r$	RUMPEP $_j$	RUMPEP $_j$ MAX $\times R$	g/d	Rumen peptides balance supply from the j th diet and lower and upper requirement constraints
	SPA		g/d	Scurf protein
	TAC		decimal ¹	True absorption coefficient
	TDN%		%	Percentage of total digestible nutrients
	TEMP1			Temperature adjustment factor for DMI
	UPA		g/d	Urinary protein
	URIN $_j$ CA		g/d	The j th group average animal's Ca requirement for urinary loss
	URIN $_j$ P		g/d	The j th group average animal's P requirement for urinary loss
	Y_i		tonne/yr	The i th annual crop yield

¹Decimal is g/g.

$$\begin{aligned}
 ME_j\text{MAX} &= ME_m + ME_p + ME_g + (\text{MILK}_j\text{MAX} \times \\
 &\quad ((0.3512 + (0.0962 \times \text{MF}))/0.644)) \\
 MP_j\text{MIN} &= MP_m + MP_p + MP_g + ((\text{MILK}_j\text{MIN} \times 10 \times \\
 &\quad \text{PP})/0.65) \\
 MP_j\text{MAX} &= MP_m + MP_p + MP_g + ((\text{MILK}_j\text{MAX} \times \\
 &\quad 10 \times \text{PP})/0.65) \\
 MP_m &= \text{SPA} + \text{UPA} + \text{FPA}
 \end{aligned}$$

Where SPA was scurf protein, and UPA was urinary protein; SPA and UPA were functions of BW and were independent of the influence of diet and milk production. However, fecal protein (FPA) was dependent on indigestible DM (IDM) and was calculated as $\text{FPA} = 0.09 \times \text{IDM}$.

The calculation of IDM in the CNCPS was a nonlinear function and was not known until the diet had been formulated. To solve this problem, an iterative procedure was used and is discussed later.

In the CNCPS, macro mineral requirements are determined using Institut National de la Recherche Agronomique (9) requirements. Some equations were modified to allow a search for the optimum rations within a range of milk production levels in lactating animals. For growing and dry animals, there was no difference between CA_jMIN and CA_jMAX and P_jMIN and P_jMAX because of zero milk production.

For lactating animals, equations in the CNCPS (5) were rewritten as:

$$\begin{aligned}
 \text{P}_j\text{MIN} &= (((22 + 0.2 \times \text{MILK}_j\text{MIN}) \times \text{BW})/100 + \\
 &\quad \text{URIN}_j\text{P} + 0.9 \times \text{MILK}_j\text{MIN} + \\
 &\quad \text{PREG}_j\text{P} + \text{GROW}_j\text{P})/\text{TAC}
 \end{aligned}$$

$$\begin{aligned}
 \text{P}_j\text{MAX} &= (((22 + 0.2 \times \text{MILK}_j\text{MAX}) \times \text{BW})/100 + \\
 &\quad \text{URIN}_j\text{P} + 0.9 \times \text{MILK}_j\text{MAX} + \\
 &\quad \text{PREG}_j\text{P} + \text{GROW}_j\text{P})/\text{TAC} \\
 \text{CA}_j\text{MIN} &= (((15 + (\text{DMI}_j\text{MIN} - 10)) \times \text{BW})/1000 + \\
 &\quad \text{URIN}_j\text{CA} + 1.2 \times (\text{MILK}_j\text{MIN} \times \\
 &\quad \text{MILK}_j\text{MINCP} \times 0.93)/3.3 + \\
 &\quad \text{PREG}_j\text{CA} + \text{GROW}_j\text{CA})/\text{TAC} \\
 \text{CA}_j\text{MAX} &= (((15 + (\text{DMI}_j\text{MAX} - 10)) \times \text{BW})/1000 + \\
 &\quad \text{URIN}_j\text{CA} + 1.2 \times (\text{MILK}_j\text{MAX} \times \\
 &\quad \text{MILK}_j\text{MAXCP} \times 0.93)/3.3 + \\
 &\quad \text{PREG}_j\text{CA} + \text{GROW}_j\text{CA})/\text{TAC}
 \end{aligned}$$

Equations to calculate N and P requirements for pregnancy, urinary excretion, growth, and the true absorption coefficients were the same as those of the CNCPS (5). Effective NDF was set to be higher than $0.2 \times \text{DMI}_j$ to maintain rumen pH. A maximum NDF constraint was set at 1 to 1.2% of NDF/kg of BW to ensure that diets did not exceed rumen capacity.

Cropland and yield. The primary focus of the cropland and yield component was to find the best combination of cropland acreage for crop production, using the equations summarized in Table 3. Users could set lower and upper acreage constraints for a crop. The lower and upper constraints on purchased feed were used to determine the amount of feeds required to be imported from off-farm sources.

Nutrient excretion. The CNCPS equations for calculating N and P excretion are difficult to convert to an LP framework, so we rearranged them for this model. It was assumed that N and P excretion was the

Table 2. Constraints and equations used in animal and feed components.

[01]	$F_{ij}\text{MIN} \leq F_{ij} \leq F_{ij}\text{MAX}$
[02]	$\text{MILK}_j\text{MIN} \times r \leq \text{MILK}_j \leq \text{MILK}_j\text{MAX} \times R$
[03]	$\text{DMI}_j\text{MIN} \times r \leq \text{DMI}_j \leq \text{SMI}_j\text{MAX} \times R$
[04]	$\text{ME}_j\text{MIN} \times r \leq \text{ME}_j \leq \text{ME}_j\text{MAX} \times R$
[05]	$\text{MP}_j\text{MIN} \times r \leq \text{MP}_j \leq \text{MP}_j\text{MAX} \times R$
[06]	$\text{RUMN}_j\text{MIN} \times r \leq \text{RUMN}_j \leq \text{RUMN}_j\text{MAX} \times R$
[07]	$\text{RUMPEP}_j\text{MIN} \times r \leq \text{RUMPEP}_j \leq \text{RUMPEP}_j\text{MAX} \times R$
[08]	$e\text{NDF}_j\text{MIN} \times r \leq e\text{NDF}_j \leq e\text{NDF}_j\text{MAX} \times R$
[09]	$\text{NDF}_j\text{MIN} \times r \leq \text{NDF}_j \leq \text{NDF}_j\text{MAX} \times R$
[10]	$\text{CA}_j\text{MIN} \times r \leq \text{CA}_j \leq \text{CA}_j\text{MAX} \times R$
[11]	$\text{P}_j\text{MIN} \times r \leq \text{P}_j \leq \text{P}_j\text{MAX} \times R$
[12]	$\text{DMI}_j\text{MIN} = ((0.0185 \times \text{BW} + 0.305(0.4 + 0.15 \times \text{PQ}) \times \text{MILK}_j\text{MIN}) \times \text{MUD1} \times \text{TEMP1}) \times \text{lag}$
[13]	$\text{DMI}_j\text{MAX} = ((0.0185 \times \text{BW} + 0.305(0.4 + 0.15 \times \text{PQ}) \times \text{MILK}_j\text{MAX}) \times \text{MUD1} \times \text{TEMP1}) \times \text{lag}$
[14]	$\text{ME}_j\text{MIN} = \text{ME}_m + \text{ME}_p + \text{ME}_g + \text{MILK}_j\text{MIN} \times ((0.3512 + (0.0962 \times \text{MF}))/0.644)$
[15]	$\text{ME}_j\text{MAX} = \text{ME}_m + \text{ME}_p + \text{ME}_g + \text{MILK}_j\text{MAX} \times ((0.3512 + (0.0962 \times \text{MF}))/0.644)$
[16]	$\text{MP}_j\text{MIN} = \text{MP}_m + \text{MP}_p + \text{MP}_g + \text{MILK}_j\text{MIN} \times 10 \times \text{PP}/0.65$
[17]	$\text{MP}_j\text{MAX} = \text{MP}_m + \text{MP}_p + \text{MP}_g + \text{MILK}_j\text{MAX} \times 10 \times \text{PP}/0.65$
[18]	$\text{P}_j\text{MIN} = ((22 + 0.2 \times \text{MILK}_j\text{MIN}) \times \text{BW})/100 + \text{URIN}_j\text{P} + 0.9 \times \text{MILK}_j\text{MIN} + \text{PREG}_j\text{P} + \text{GROW}_j\text{P})/\text{TAC}$
[19]	$\text{P}_j\text{MAX} = ((22 + 0.2 \times \text{MILK}_j\text{MAX}) \times \text{BW})/100 + \text{URIN}_j\text{P} + 0.9 \times \text{MILK}_j\text{MAX} + \text{PREG}_j\text{P} + \text{GROW}_j\text{P})/\text{TAC}$
[20]	$\text{CA}_j\text{MIN} = (((15 + (\text{DMI}_j\text{MIN} - 10)) \times \text{BW})/1000 + \text{URIN}_j\text{CA} + 1.2 \times (\text{MILK}_j\text{MIN} \times \text{MILK}_j\text{MINCP} \times 0.93)/3.3 + \text{PREG}_j\text{CA} + \text{GROW}_j\text{CA})/\text{TAC}$
[21]	$\text{CA}_j\text{MAX} = (((15 + (\text{DMI}_j\text{MAX} - 10)) \times \text{BW})/1000 + \text{URIN}_j\text{CA} + 1.2 \times (\text{MILK}_j\text{MAX} \times \text{MILK}_j\text{MAXCP} \times 0.93)/3.3 + \text{PREG}_j\text{CA} + \text{GROW}_j\text{CA})/\text{TAC}$

Table 3. Constraints and equations used in crop land and yield, nutrient excretion and economic parameters.

Crop Land and Yield	
[22]	$HL = \sum_{i=1}^m L_i$
[23]	$HL_i Y_i = \sum_{i=1}^m L_i \times Y_i$
[24]	$HF_i BUY = HF_i - (1 - LOSS_i) \times HL_i Y_i$
[25]	$L_{MIN} \leq L_i \leq L_{iMAX}$
[26]	$HF_i = 365 \times (1 + ORT) \times \sum_{j=1}^n F_{ij} \times G_j$
[27]	$HF_i BUY_{MIN} \leq HF_i BUY \leq HF_i BUY_{MAX}$
Nutrient Excretion	
[28]	$EXEP_j = \sum_{i=1}^m F_{ij} \times F_i P / 6.25 - MILK_j \times 0.9 - GROW_{jP} - PREG_{jP} - RESERVE_{jP}$
[29]	$EXEN_j = \sum_{i=1}^m F_{ij} \times F_i CP - MILK_j CP \times (10/6.38) - GROW_{jN} - PREG_{jN} - RESERVE_{jN}$
[30]	$HEXEN = \sum_{j=1}^n EXEN_j \times G_j$
[31]	$HEXEP = \sum_{j=1}^n EXEP_j \times G_j$
[32]	$HEXEN_{MIN} \leq HEXEN \leq HEXEN_{MAX}$
[33]	$HEXEP_{MIN} \leq HEXEP \leq HEXEP_{MAX}$
Economic Parameters	
[34]	$HF\$ = \sum_{i=1}^m HF_i \times F_i\$ + \sum_{i=1}^m (HF_i - HL_i Y_i) \times F_i\$\$$
[35]	$HMILK\$ = \sum_{j=1}^n MILK_j \times MILK\$$
[36]	$HMEAT\$ = \sum_{j=1}^n HMEAT_j \times MEAT\$$
[37]	$HFER\$ = \sum_{i=1}^m FER_i \times FER_i\$$
[38]	$HPROFIT = HMILK\$ + HMEAT\$ - HF\$ - HFER\$$
[39]	$HPROFIT_{MIN} \leq HPROFIT \leq HPROFIT_{MAX}$

DMI of N and P minus N and P retained as tissue or milk. The equations (Table 3) were:

$$EXEP_j = 1000 \times DMI_{jP} - MILK_j \times 0.9 - GROW_{jP} - PREG_{jP} - RESERVE_{jP}$$

$$EXEN_j = 1000 \times DMI_{jN} - MILK_j \times MILK_j CP \times 10/6.38 - GROW_{jN} - PREG_{jN} - RESERVE_{jN},$$

where $GROW_{jP}$, $PREG_{jP}$, $GROW_{jN}$, and $PREG_{jN}$ were animal-dependent factors independent of the influence of diet composition; therefore, they could be imported directly from CNCPS calculations and did not have to be rewritten. Body reserve P ($RESERVE_{jP}$) was a negative value when supply was less than the requirement and was zero when supply was more than the requirement, but we normally balanced the diets above

the requirement; therefore, $RESERVE_{jP}$ could be set to zero. Body reserve N ($RESERVE_{jN}$) was a nonlinear function that required an iterative procedure to approach an approximate value.

Economic parameters. Return over feed costs was calculated as the sale of milk and meat minus feed price and fertilizer cost [Equation 38]. Price differences between homegrown and purchased resources were accounted for in this model [Equation 34]. Lower and upper constraints on profit could be set to attain the desired return over feed cost.

Nitrogen and phosphorus mass nutrient balance. Table 4 lists the equations in the N and P mass nutrient balance component. Nitrogen balance was calculated as the amount of imported N (N-fixation, N in purchased feed and fertilizer) less exported N in milk, meat, and crops sold (10). Nitrogen fixation was assumed to be 60% of the amount of protein produced

from legume fields (those with more than 90% legume), with the balance coming from soil organic matter, manure N, or fertilizer N (12). Miller and Heichel (14) used a similar calculation to estimate N fixation. The P and K balances were determined similarly, except that there was no fixation of these nutrients. Lower and upper constraints on the N, P balance could be set to attain the balance level to be achieved.

Objective Functions

In formulating the rations for the whole herd, the foremost objective was to identify the combination of feeds that maximized return over feed costs for the farm while satisfying the imposed constraints. However, given the nature of the Microsoft Excel Solver add-in, the objective function of this model could also be easily changed to any targeted spreadsheet cells such as annual N or P excretion, annual milk production, and whole farm N or P balance, depending on the user's objectives.

Iterative Procedure to Approximate Nonlinear Functions

In the CNCPS, passage rate is a function of BW (kg), DMI (kg/d), and forage (% DM); rumen pH value is a function of eNDF; N recycled in the rumen is a function of CP (%DM); MP_m is influenced by TDN (%DM); and microbial yield from nonfiber carbohydrate (NFC) bacteria (g of bacteria/g of NFC carbohydrate) is influenced by the $NFC:(NFC + PEPTIDE)$. These values [MP_m , N recycling, pH, passage rate, $NFC:(NFC + PEPTIDE)$] were required to calculate feed biological value (BV) and requirements in the CNCPS, but they were nonlinear functions and could not be used in the LP matrix. To balance a ration for a group, therefore, the first step was to compute expected DMI and biological values (TDN, eNDF, CP%, forage%, $NFC:[NFC + PEPTIDE]$) for feeds to be considered for the ration. The model could then use these biological values for each feed in a LP matrix to generate the first optimum diet that met the constraints set. After the first optimum diet was formulated, it was evaluated in the

Table 4. Constraints and equations used in mass nutrient balance components.

Nitrogen coefficients	
[40]	$HL_i Y_i NFIX = \sum_{i=1}^m L_i \times Y_i \times F_i CP \times 0.6/6.25$ (if L_i is legume field)
[41]	$HFBUYN = \sum_{j=1}^n HF_j \times F_j CP/6.25$
[42]	$HFERN = \sum_{i=1}^m FER_i \times FER_i N$
[43]	$HMILKN = HMILK \times MILKCP/6.38$
[44]	$HMEATN = HMEAT \times MEATCP/6.25$
[45]	$BALN = HL_i Y_i NFIX + HFBUYN + HFERN - HMILKN - HMEATN$
[46]	$BALNMIN \leq BALN \leq BALNMAX$
Phosphorus coefficients	
[47]	$HFBUYP = \sum_{j=1}^n HF_j \times F_j P$
[48]	$HFERP = \sum_{i=1}^m FER_i \times FER_i P$
[49]	$HMILKP = HMILK \times MILKP$
[50]	$HMEATP = HMEAT \times MEATP$
[51]	$BALP = HFBUYP + HFERP - HMILKP - HMEATP$
[52]	$BALPMIN \leq BALP \leq BALPMAX$
Potassium coefficients	
[53]	$HFBUYK = \sum_{j=1}^n HF_j \times F_j K$
[54]	$HFERK = \sum_{i=1}^m FER_i \times FER_i K$
[55]	$HMILKK = HMILK \times MILKK$
[56]	$HMEATK = HMEAT \times MEATK$
[57]	$BALK = HFBUYK + HFERK - HMILKK - HMEATK$

CNCPS to ensure that optimal diets were consistent with the animal nutrient requirements and feed BV. The model developed by Tedeschi et al. (18) does not require the $NFC:(NFC + PEPTIDE)$ relationship because the NFC bacteria improvement was not optimized.

Animal nutrient requirements and the BV of feeds vary depending on the composition of the diet, environmental conditions, and animal characteristics (5). However, variation in feed BV and animal nutrient requirements with diet composition was particularly troublesome in a LP formulation because it implied violation of the standard assumption of invariant nutritional coefficients for the LP matrix (14). Tedeschi et al. (18) used an iterative procedure between CNCPS and the LP model to ensure that optimal diets were consistent with the animal nutrient requirements and feed BV. In this model, initial nutrient requirements and BV were determined with the CNCPS, then they were modified based on the optimal diet composition selected by the LP model for each group of animals. This procedure was repeated, and the diet was evaluated until diet composition (hence BV and nutrient requirements) predicted by the LP model changed little from one iteration to the next. Figure 1 summarizes this procedure.

Computer Techniques

This LP optimization model was developed with the use of Microsoft Excel Solver add-in, a user-friendly LP function added to Excel. This program was operated with Microsoft Windows 95 (Microsoft Corp., Seattle, WA) on an IBM-PC compatible system with 16 MB of RAM.

Whole-Farm Evaluation

Data from the case study farm described in Wang et al. (21) was used to demonstrate how the optimization procedure was applied. The herd feed analysis and initial rations for the case study farm was described by Tedeschi et al. (18). These data were the required inputs in the CNCPS to calculate animal requirements and feed biological values in this model. The optimized diet of the model presented by Tedeschi et al. (18) is slightly different from the model presented here mainly because of the constraints structure.

The yields of corn silage (CS), alfalfa silage (MMLa), grass hay (GHa), and high moisture shell corn (HMSC) were 6.9, 4.7, 5.0, and 3.0 tonne/ha (DM basis), respectively, during 1996. Two hundred hectares were used for hay crops and 230 ha for CS and HMSC. Forage production totals (DM basis) were 874, 1346,

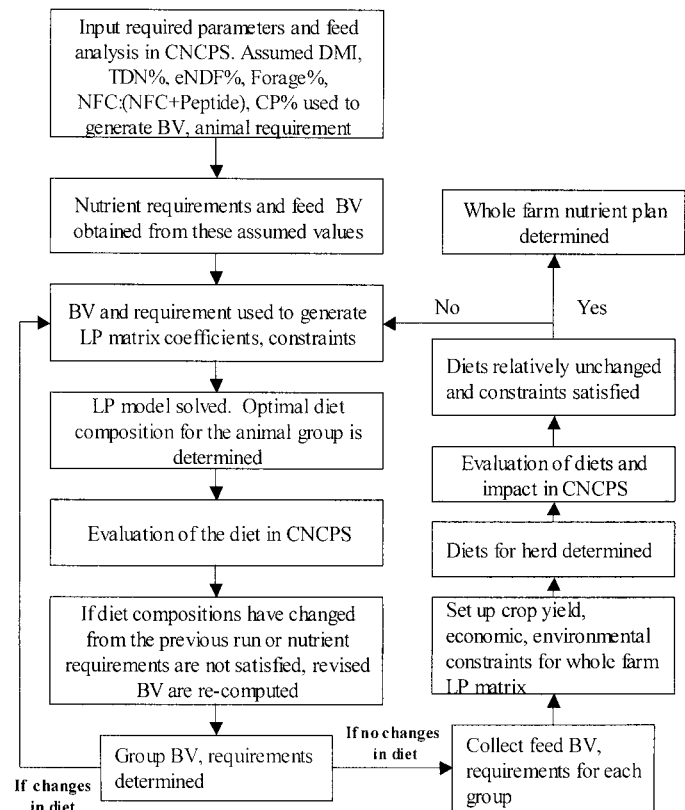


Figure 1. Iterative procedure for developing diets for the linear programming model.

120, and 75 tonne for MMLa, CS, GHa, and HMSC, respectively. After accounting for the 20 and 24% DM loss in MMLa and CS during processing and storage, approximately 699 and 1023 tonne for MMLa and CS, respectively, were available for feeding or for sale. The annual milk production for 1996 was 3888 tonne. For this example, we evaluated how well the feed produced was allocated to match the animal requirements, how the rations could be adjusted, and what the economic and environmental impact of the rebalanced rations would be. In the rebalancing process, we kept the crop acreage, crop yield, and milk production the same and assumed the herd had to consume all homegrown crops up to their requirements.

RESULTS AND DISCUSSION

The impact of the optimization process on feed allocation to various herd groups, and nutrient mass balance is presented in Tables 5 and 6. Table 5 shows the annual feed consumption of each group in the original and rebalanced scenarios. This table shows that in the original diet, most of the alfalfa silage production (MMLa) was allocated to lactating cow groups (70%);

Table 6. Mass balance (tonne/yr) with original and rebalanced forages.

	Original			Rebalanced		
	N	P	K	N	P	K
N fixation	15.1	15.1
Fertilizer	4.2	5.7	5.4	4.2	5.7	5.4
Feeds	60.9	6.8	20.4	53.4	6.2	18.9
Total Imports	80.2	12.5	25.7	72.7	11.8	24.2
Milk	19.0	3.9	5.8	19.0	3.9	5.8
Meat ¹	3.1	0.9	0.2	3.1	0.9	0.2
Feeds	6.4	1.0	3.5	5.9	0.9	3.4
Total Exports	29.5	5.7	9.4	28.0	5.7	9.4
Remaining	51.7	6.7	16.2	44.7	6.1	14.8
%Remaining	64.5	54.0	63.0	61.5	51.7	61.2

¹Total: 121 tonne; 2.53, 0.72, 0.19% of live weight for N, P and K, respectively as reported by Klausner (12).

tion in temperature on DMI was ignored. Normally, in winter the animals consume more DM than in summer (2, 8). We can further divide the simulation period into months or seasons to account for this. In this model, we assumed the fertilizer application rate of a crop was a constant; however, the fertilizer application rate was according to the manure application and crop rotation. New versions of this model will include a more detailed function to account for the manure availability and crop rotation to make it a more complete whole farm nutrient model.

The described LP optimization model can be applied in a number of ways: 1) to help animal nutritionists evaluate rations for meeting the requirements of each group of animals while matching forage and purchased feed inventories; 2) to help farmers design crop production plans that best match animal requirements; and 3) to evaluate the environmental and financial impacts of different feeding strategies.

APPLICATION

The CNCPS can be used in an LP structure to allocate feeds across groups in the herd in designing a whole farm nutrient management plan. The equations and tables presented illustrate how to structure the CNCPS model for this purpose. The model uses animal, environmental, and feed composition information that can be determined for each production situation.

Environmental protection, profit considerations, and animal nutrient requirements may be conflicting objectives. However, the optimization model can allocate forages and balance rations for different production groups. This forage allocation procedure maximizes returns over feed costs while minimizing nutrient excretion. The model results show that optimizing the feeding strategy has the potential to decrease nutrients retained on farm, which demonstrates that en-

vironmental protection can be addressed simultaneously with profitability on an operating dairy farm. The optimization model is a useful tool in identifying environmentally and economically feasible solutions when designing a whole farm nutrient management plan.

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