Whole-Herd Optimization with the Cornell Net Carbohydrate and Protein System. I. Predicting Feed Biological Values for Diet Optimization with Linear Programming[1]

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ABSTRACT

We developed a diet optimizer for least-cost diet formulation with the Cornell Net Carbohydrate and Protein System (CNCPS) using linear programming. The CNCPS model is intrinsically nonlinear, and feed biological values vary with animal and feed characteristics. To allow linear diet optimization, we first used the CNCPS model to generate biological values to characterize the energy and protein content of each feed for the specific group for which the diet was being formulated. The biological values used were metabolizable energy (Mcal/kg), metabolizable protein [(% dry matter (DM)], passage rate (%/h), bacteria yield efficiencies (g/g), and degradation rate of the carbohydrate B2 fraction (%/h). In addition, the ruminal balances for nitrogen and peptides were included in the optimizer to optimize ruminal degradation of fiber. The objective function was to minimize diet cost subject to animal requirement and feed availability constraints. The animal constraints were set by requirements for DM intake (kg/d), metabolizable energy (Mcal/kg), metabolizable protein (%DM), and effective neutral detergent fiber (%DM) for a given level of production. Data from a dairy farm were used to evaluate this linear diet optimizer. Across all classes of dairy cattle, the CNCPS 4.0 model typically obtained a solution in less than six iterations that met the requirements with nearly 100% accuracy. We conclude this linear optimizer can be used to accurately formulate least-cost diets with the CNCPS model.

(**Key words:** diet optimization, least cost formulation, linear programming)

Abbreviation key: CNCPS = Cornell Net Carbohydrate and Protein System, **CPM Dairy** = Cornell-Penn-Minor Dairy software, **CuNMPS** = Cornell University Nutrient Management Planning System, **eNDF** = efficient NDF, **IPM** = interior point methods, $_j$ = each feed, **kd** = digestion rate, **kp** = passage rate, **ME** = metabolizable energy, **MP** = metabolizable protein, **Y** = yield efficiency. Also see Table 1.

INTRODUCTION

Ration optimization routines have been developed with the objective of enhancing cattle production through accurate and rapid least-cost ration formulation, with use of linear programming techniques (28). Linear programming is a mathematical system in which the objective function is linear in the unknowns, and the constraints also consist of linear equalities or inequalities or both (18). In a canonical vector notation, a linear problem has the following aspect:

$$\text{minimize: } \mathbf{C}^{\mathbf{T}}\mathbf{x}$$
$$\text{subject to: } \mathbf{Ax} = \mathbf{B} \text{ and } x \geq 0$$

Where **x** is an n-dimensional column vector in which each component is positive, $\mathbf{C}^{\mathbf{T}}$ is an n-dimensional row vector, **A** is an m × n matrix, and **B** is an m-dimensional column vector.

Two methods have been used to solve linear programming problems: boundary methods and interior point methods (**IPM**). Boundary methods comprise primal and dual simplex methods. Interior point methods include several methods; the primal affine scaling method is the most commonly used (27). One of the most commonly used mathematical methods to solve linear programs is the simplex method. The concept of this method is to proceed from one basic feasible solution (that is, one extreme point) of the constraint set of the problem in standard form to another. In this way, simplex methods generate a sequence of points on the boundary of the polyhedron. This continuous search is performed in such a way as to decrease the value of the objective function until a minimum is reached. In contrast, IPM generate a sequence of points in the interior of the polyhedron. These methods may be advantageous for large problems (27).

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Several papers have discussed ration formulation models based on linear programming either for dairy cattle (23, 25), beef cattle (5, 11), or dual-purpose cattle (21, 22) applications. Grazing applications have also been investigated (4, 10). The use of nonlinear programming has been limited due to the local optimum solutions, which are nonoptimum solutions (12). But with the advances in computer technology and development of more powerful algorithms, nonlinear programming has also been used in ration formulation for beef cattle (14) and for dairy cattle (**CPM Dairy**; Cornell-Penn-Minor Dairy software version 1.0 manual released in 1998). The combination of linear and nonlinear algorithms has also been used as a new approach to solve nonlinear problems in dairy (13) and feedlot (3) applications.

The Cornell Net Carbohydrate and Protein System (**CNCPS**) model has been used for both dairy cattle (6, 7, 17) and beef cattle (20) applications. More recently, the CNCPS has been expanded from a simple group diet evaluation to annual whole herd nutrient evaluation as part of a program developed for use in whole farm nutrient management (Cornell University Nutrient Management Planning System, **CuNMPS**). The CuNMPS includes soil fertility, crop and manure nutrient management, crop rotations, and herd nutrition considerations (2, 15, 16, 31). The role of the CNCPS in the CuNMPS model is to optimize diets for all groups in the herd and to accurately predict annual herd feed requirements and nutrient excretion (31).

The objective of the study reported here was to develop a linear programming procedure for the CNCPS model to optimize diets for least cost within each group. To evaluate the optimizer robustness, accuracy, and usability, we analyzed the diet optimization for lactating and dry cows and for replacement heifers.

MATERIALS AND METHODS

A description of variables used to develop the CNCPS version 4.0 optimizer is shown in Table 1, and a flowchart of the optimization steps is given in Figure 1.

Calculation of Biological Values

The behavior of the CNCPS model is intrinsically nonlinear because the feed biological values vary with animal and feed characteristics and their interactions. However, the CNCPS model can be used to generate specific values used to predict the metabolizable energy and protein content of each feed in a given diet. These values in turn can be used as coefficients in a linear matrix. If this process is continually repeated (iteration) until variation in animal response and diet change is within an acceptable deviation, an optimal diet can be obtained for that specific situation.

The utilization of an initial diet to achieve these coefficients for each feed is important because the nutritional value of the diet depends on the interaction between rates of degradation and rates of passage of feeds, feed composition, and intake (32). These degradation and passage rates for each feed change depending on their concentrations in the diet (20).

Step 1. The first step for the optimization procedure was to enter the inputs required by the CNCPS (animal characteristics, management practices, environmental aspects, feed composition, and DMI) for the group to estimate the biological values of each feed to be considered in that diet as described by Fox et al. (7).

Step 2. This step computed metabolizable energy and protein values for each feed with the CNCPS model for the feeding group described. Table 2 contains the equations used in the CNCPS model to compute carbohydrate and protein fractions and their ruminal degradability (29). These equations are used in the optimizer as well as those described in Table 3, which shows all the equations and constraints used by the optimizer. The calculation of biological values by the CNCPS has been discussed elsewhere (8, 9, 20, 24, 26, 29).

The optimizer uses, for each feed ($_j$), metabolizable energy (**ME$_j$**, Mcal/kg), metabolizable protein (**MP$_j$**, %DM), passage rate (**kp$_j$**, %/h), yield efficiency of fiber, sugar, and carbohydrate B1 digester bacteria (**Y$_{1j}$**, **Y$_{2j}$**, and **Y$_{3j}$**, g of bacteria/g of carbohydrate digested, respectively), and carbohydrate B2 fraction degradation rate (**kd$_{6j}$**, %/h) adjusted for ruminal pH. Metabolizable protein (%DM) comprised the three feed protein fractions that escaped ruminal degradation and were intestinally absorbed (REPB1$_j$, REPB2$_j$, and 0.8×REPB3$_j$) plus the bacterial true protein (REBTPJ) produced by each feed degraded in the rumen as in Table 2. The bacterial growth equations were those described by Russell et al. (26), NRC (20), and Tedeschi et al. (30).

With the purpose of obtaining these biological values for the first optimization only, a standard diet (a sample of a "standard diet" to initialize the optimizer would be 10 kg of alfalfa silage and 10 kg of orchardgrass hay) having effective NDF (**eNDF**; %DM) higher than 20% was entered in the CNCPS model to compute the maximum microbial yield (g/g of DM for each feed). Then, to generate the biological values, we entered each feed in this "standard diet" at a very low amount (0.01 kg), which permitted the computation of biological values without affecting the diet composition significantly. After the first optimization, more accurate biological values were estimated with the ingredients of the current diet as a "standard diet." The current diet could

Table 1. Description of the acronyms used in the optimization algorithm.

Acronyms	Description
ADF	Acid detergent fiber, %DM
ADFIP$_j$	Insoluble protein in the ADF for each feed, %CP
ASH$_j$	Ash for each feed, %DM
Ca	Calcium, g/d
CA$_j$	CHO A fraction (sugars and organic acids) for each feed, %DM
CB1$_j$	CHO B1 fraction (starch and soluble fibers) for each feed, % DM
CB1NFC$_j$	CHO B1 fracation (starch and soluble fibers) for each feed, %NFC
CB2$_j$	CHO B2 fraction (available NDF) for each feed, %DM
CC$_j$	CHO C fraction (indigestible) for each feed, %DM
CC$_j$	CHO C fracation (indigestible) for each feed, %DM
CHO$_j$	Carbohydrate for each feed, %DM
CP$_j$	CP for each feed, %DM
DMI$_j$	DMI for each feed, g/d
FAT$_j$	Fat for each feed, %DM
Lignin$_j$	Lignin for each feed, %NDF
NDFIP$_i$	Insoluble protein in the NDF for each feed, %CP
NDF$_j$	NDF for each feed, %DM
NFC	Nonfiber carbohydrate, %DM
NPN$_j$	Nonprotein nitrogen for each feed, %SolCP
P	Phosphorus, g/d
PA$_j$	Protein A fraction for each feed, %DM
PB1$_j$	Protein B1 fraction for each feed, %
PB2$_j$	Protein B2 fraction for each feed, %
PB3$_j$	Protein B3 fraction for each feed, %
PC$_j$	Protein C fraction for each feed, % DM
RDCA$_j$	Ruminally degraded CA for each feed, g/d
RDCB1$_j$	Ruminally degraded CB1 for each feed, g/d
RDCB2$_j$	Ruminally degraded CB2 for each feed, g/d
RDPA$_j$	Ruminally degraded PA for each feed, g/d
RDPB1$_j$	Ruminally degraded PB1 for each feed, g/d
RDPB2$_j$	Ruminally degraded PB2 for each feed, g/d
RDPB3$_j$	Ruminally degraded PB3 for each feed, g/d
REBTP$_j$	Ruminally escaped bacterial true protein for each feed, g/d
REPB1$_j$	Ruminally escaped protein B1 fraction for each feed, g/d
REPB2$_j$	Ruminally escaped protein B2 fraction for each feed, g/d
REPB3$_j$	Ruminally escaped protein B3 fracation for each feed, g/d
REP$_j$	Ruminally escaped protein for each feed, g/d
RNAvail	Ruminal N available, g/d
RNReq	Ruminal N required, g/d
RPepAvail	Ruminal peptide available, g/d
RPepReq	Ruminal peptide required, g/d
SolCP$_j$	Soluble CP for each feed, %CP

be used as a "standard diet" only if the combination of feeds satisfied the eNDF restriction.

In addition to these biological values, the optimizer also created a set of constraints using eNDF$_j$ (%DM) to ensure adequate ruminal pH, and for each mineral of interest.

The optimizer computed the ruminal ammonia and peptide concentration necessary to meet the bacteria requirements, as follows: 1) the ruminal nitrogen required (**RNReq**, equation 8) to support the ruminal bacteria growth was the sum of ruminal degraded carbohydrates (**RDCA$_j$**, **RDCBl$_j$**, and **RDCB2$_j$**; Table 2) multiplied by their respective bacteria yield efficiency (**Y2$_j$**, **Y3$_j$**, and **Y1$_j$**), which was then multiplied by 0.625 (bacteria %CP content) to estimate the total bacteria protein; 2) the ruminal nitrogen available (**RNAvail**, equation 9) was computed, including nonprotein nitrogen (**RDPA$_j$**) and peptides from ruminal true protein degradation (**RDPBl$_j$**, **RDPB2$_j$**, and **RDPB3$_j$**) (Table 2). The ruminal nitrogen balance was the difference between RNAvail and RNReq; and 3) the ruminal peptide balance was computed as the difference between ruminal degraded peptides (**RPepAvail**, equation 11) and the peptides utilized by nonfiber carbohydrate bacteria (**RPepReq**, equation 10).

Step 3. This step set the constraints based on the animal requirements (DMI$_r$, ME$_r$, MP$_r$, eNDF$_r$, Ca$_r$, and P$_r$; Equations 12 to 17) and the objective function. The objective function was a linear equation with which the linear programming minimized (or maximized) the result. The objective function in this optimizer was to minimize the diet cost. If several scenarios were evaluated (e.g., different levels of milk production) and a curve using the cost versus profit (income minus feed costs) was drawn, the maximum profit diet could be inferred by the lowest point of the curve.

The optimizer utilized ranges (minimum and maximum percentage of the requirement) for each constraint. Minimum nutrient constraints were set to ensure the optimized diet met the animal requirements. They could be set at greater than 100% to add a safety factor. Maximum constraints were set at the lowest maximum that allowed a feasible solution and the highest acceptable maximum to avoid unacceptable levels of overfeeding based either on feed availability or feeding recommendations. Our constraints were set at the minimum required to ensure the CNCPS predicted requirements were met and the lowest maximum that would allow a feasible solution.

Minimum and maximum constraints were set for DMI, ME, MP, Ca, P, RNBal, and RPepBal. The minimum constraint for ruminal N balance (RNBal) was set below 100% of required ruminal N because the recycled N was not accounted for due to the nonlinearity of the equation. In addition to these constraints, upper and lower limitations for the amount of each feed were used.

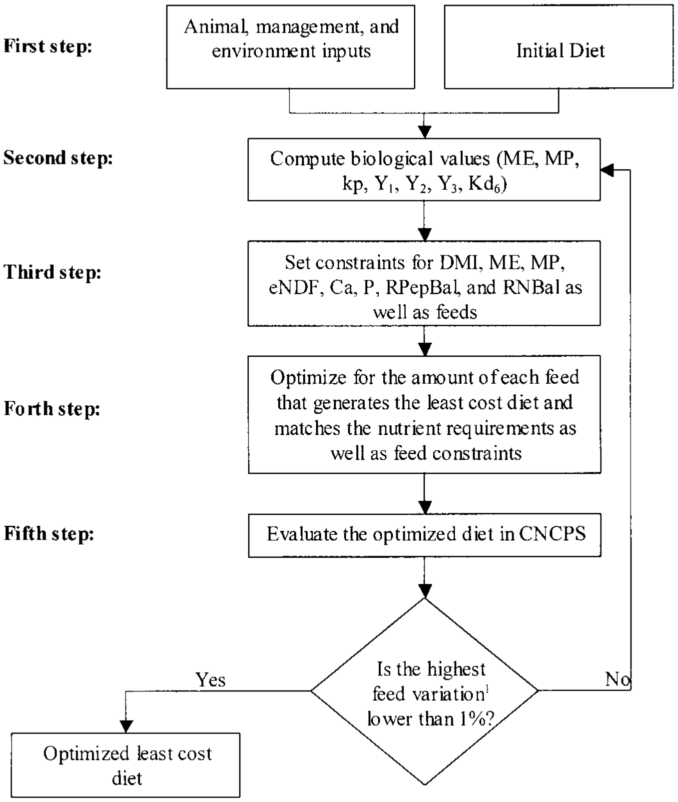

Figure 1. Optimizer flowchart showing the optimization steps.

Step 4. This step optimized the diet. The amount of each feed was determined by the best proportion of each feed that generated the least diet cost to meet a set of constraints (DMI; requirements for energy, protein, and minerals; ruminal adequate balance for ammonia and peptides; and constraints set for feeds; equation 20).

Step 5. The optimized diet was reevaluated by the CNCPS model. The diet was reoptimized until it met the constraints set for requirements.

Figure 1 shows the optimization flowchart, indicating the five steps required to obtain the least-cost diet that met the requirements with the structure of the CNCPS. The flowchart showed the diet being rebalanced until the highest variation (changes in inclusion in kg of DM) for any feed between consecutive optimized diets was lower than 1%. This limit of 1% can be changed to have less or more iterations with lower or higher accuracy in diet formulation, respectively.

Assumptions and Restrictions

In general, the following optimizer limitations were due to the consequence of the linearization procedure used to overcome the nonlinearity of the CNCPS computed biological values:

1. Equations 9 and 11 assumed that all ruminal-degraded protein (peptides) was available for microbial growth. A small amount of this degraded protein is likely to escape the rumen if the rate of peptide uptake is lower than the rate of degradation, however. Hence, the balance assumed by the optimizer may not be exactly the same as that with the CNCPS, depending on the microbial growth, which was dependent on the diet composition.
2. The equation to account for recycled N had nonlinear behavior that restricted its utilization in a linear programming algorithm. Consequently, this N was available in the rumen in addition to that from the diet. We addressed this limitation by setting the minimum constraint for ruminal N balance lower than 100% of that required by microbial growth (equation 18) to allow for the recycled N that would be added to the supply of N when the optimized diet was evaluated by the CNCPS model in step 5.
3. The microbial yield (Y_2 and Y_3) is adjusted by the ratio between degraded peptide and total degraded carbohydrate. Once the diet changes, it is likely that this ratio will change; consequently changes in microbial yield occurred, and the biological values will not be the same.
4. The passage rate may vary, depending on the diet composition, which would change all the biological values for each feed.

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[1] Feed variation is the change in inclusion of any feed in kilograms of DM between previous and current optimized diet.

Table 2. Carbohydrate and protein fractions and their ruminally-degraded amount.[1]

CHO and Protein fractions	Ruminal degraded fractions
$CHO_j = 100 - CP_j - FAT_j - ASH_j$	$RDCA_j = DMI_{oj} \times CA_j \times \left(\frac{kd_{4j}}{kd_{4j} + kp_j}\right)$
$CC_j = \frac{NDF_j \times Lignin_j \times 2.4}{100}$	$RDCB1_j = DMI_{oj} \times CB1_j \times \left(\frac{kd_{5j}}{kd_{5j} + kp_j}\right)$
$CB2_j = \left(NDF_j - \frac{NDFIP_j \times CP_j}{100}\right) - CC_j$	$RDCB2_j = DMI_{oj} \times CB2_j \times \left(\frac{kd_{6j}}{kd_{6j} + kp_j}\right)$
$CB1_j = CB1NFC_j \times \left(\frac{CHO_j - CB2_j - CC_j}{100}\right)$	$RDPA_j = DMI_{oj} \times PA_j$
$CA_j = CHO_j - CB2_j - CC_j - CB1_j$	$RDPB1_j = DMI_{oj} \times PB1_j \times \left(\frac{kd_{1j}}{kd_{1j} + kp_j}\right)$
$PA_j = \frac{NPN_j \times SolCP_j \times CP_j}{10{,}000}$	$RDPB2_j = DMI_{oj} \times PB2_j \times \left(\frac{kd_{2j}}{kd_{2j} + kp_j}\right)$
$PB1_j = \frac{SolCP_j \times CP_j}{100} - PA_j$	$RDPB3_j = DMI_{oj} \times PB3_j \times \left(\frac{kd_{3j}}{kd_{3j} + kp_j}\right)$
$PC_j = \frac{ADFIP_j \times CP_j}{100}$	$REPB1_j = DMI_{oj} \times PB1_j \times \left(\frac{kp_j}{kd_{1j} + kp_j}\right)$
$PB3_j = \frac{(NDFIP_j - ADFIP_j) \times CP_j}{100}$	$REPB2_j = DMI_{oj} \times PB2_j \times \left(\frac{kp_j}{kd_{2j} + kp_j}\right)$
$PB2_j = CP_j - (PA_j + PB1_j + PB3_j + PC_j)$	$REPB3_j = DMI_{oj} \times PB3_j \times \left(\frac{kp_j}{kd_{3j} + kp_j}\right)$
	$REBTP_j = 0.625 \times 0.6 \times (BacterialYield_j)$ $REP_j = REPB1_j + REPB2_j + 0.8 \times REPB3_j$ $MP_j(\%DM) = \frac{REP_j + REBTP_j}{DMI_j}$

[1] Subscript "o" means optimized value and "j" means the jth feed.

Evaluation of the Optimizer

The performance of the CNCPS version 4.0 linear optimizer was evaluated with each group in the case study described by Wang et al. (34), which was used to develop the whole-herd forage allocation procedure described in the companion paper (33). Table 4 summarizes the inputs used to describe each group in the herd for the evaluation. The composition of feeds available for diet formulation is shown in Table 5.

The number of iterations was the number of simulations performed until the change in diet composition was less than 1% (precision of the optimization).

Computer Technique

All simulations were executed on a PC microcomputer, with an Intel Pentium II processor at 333 MHz and 64 Mb of RAM running Microsoft Windows 98 (Microsoft Corp., Seattle, WA). The CNCPS 4.0 (9) in spreadsheet format was used to evaluate the optimizer performance. In this version, the linear programming was performed using the simplex method included in the MS Excel 2000 Solver add-in.

RESULTS AND DISCUSSION

The CNCPS evaluation of the nonoptimized diets used for each group is described in Table 6. This evaluation indicated that several groups had excess of dietary ME and MP, and two groups were deficient in ruminal N. Tables 7, 8, and 9 summarize the optimized diet composition for lactating and dry cows, and replacement heifers, and Table 10 has the minimum and maximum constraints used to formulate the diets to meet the nutrient requirements.

Lactating Cow Groups

The optimized diets for lactating cows are shown in Table 7. On average, the optimizer converged to an optimum diet in less than seven iterations. This is higher than the number of iterations found by Nicholson et al. (22). However, in that paper a limit of 5% in diet variation from the previous iteration was used instead of 1% to reach the final diet. A limit of 5% would decrease the number of iterations.

The minimum and maximum set to optimize the diets for DMI, ME, and MP constraints were 100% of the

Table 3. Linear programming equations used by the CNCPS model optimizer.[1]

$$[01]\quad \text{Minimize } Z = \sum_{j=1}^{n}(\text{Feed}_j \times \text{Cost}_j)$$

$$[02]\quad \text{DMI}_o = \sum_{j=1}^{n}(\text{Feed}_j)$$

$$[03]\quad \text{ME}_o = \sum_{j=1}^{n}(\text{Feed}_j \times \text{ME}_j)$$

$$[04]\quad \text{MP}_o = \sum_{j=1}^{n}(\text{Feed}_j \times \text{MP}_j)$$

$$[05]\quad \text{eNDF}_o = \sum_{j=1}^{n}(\text{Feed}_j \times \text{eNDF}_j)$$

$$[06]\quad \text{Ca}_o = \sum_{j=1}^{n}(\text{Feed}_j \times \text{Ca}_j)$$

$$[07]\quad \text{P}_o = \sum_{j=1}^{n}(\text{Feed}_j \times \text{P}_j)$$

$$[08]\quad \text{RNReq} = \sum_{j=1}^{n}(\text{RDCA}_j \times Y_{2j} + \text{RDCB1}_j \times Y_{3j} + \text{RDCB2}_j \times Y_{1j}) \times 0.625$$

$$[09]\quad \text{RNAvail}_o = \sum_{j=1}^{n}(\text{RDPA}_j + \text{RDPB1}_j + \text{RDPB2}_j + \text{RDPB3}_j)$$

$$[10]\quad \text{RPepReq} = 0.66 \times \left[\sum_{j=1}^{n}(\text{RDCA}_j \times Y_{2j} + \text{RDCB1}_j \times Y_{3j}) \times 0.625\right]$$

$$[11]\quad \text{RPepAvail}_o = \sum_{j=1}^{n}(\text{RDPB1}_j + \text{RDPB2}_j + \text{RDPB3}_j)$$

$$[12]\quad \text{Minimum} \times \text{DMI}_r \le \text{DMI}_o \le \text{Maximum} \times \text{DMI}_r$$
$$[13]\quad \text{Minimum} \times \text{ME}_r \le \text{ME}_o \le \text{Maximum} \times \text{ME}_r$$
$$[14]\quad \text{Minimum} \times \text{MP}_r \le \text{MP}_o \le \text{Maximum} \times \text{MP}_r$$
$$[15]\quad \text{Minimum} \times \text{eNDF}_r \le \text{eNDF}_o \le \text{Maximum} \times \text{eNDF}_r$$
$$[16]\quad \text{Minimum} \times \text{Ca}_r \le \text{Ca}_o \le \text{Maximum} \times \text{Ca}_r$$
$$[17]\quad \text{Minimum} \times \text{P}_r \le \text{P}_o \le \text{Maximum} \times \text{P}_r$$
$$[18]\quad \text{Minimum} \times \text{RNReq} \le \text{RNAvail}_o \le \text{Maximum} \times \text{RNReq}$$
$$[19]\quad \text{Minimum} \times \text{RPepReq} \le \text{RPepAvail}_o \le \text{Maximum} \times \text{RPepReq}$$
$$[20]\quad \text{FeedMin}_j \le \text{Feed}_{jo} \le \text{FeedMax}_j$$

[1] Subscript “r” means required, “o” means optimized, and “j” means the jth feed.

required (Table 10). The constraints required for ruminal N minimum and maximum to allow for recycled N in meeting the ruminal N requirement varied from 80 to 85% for high cows, 80 to 90% for medium cows, and 75 to 90% for low cows. These values suggested that recycled N provides 10 to 25% of the ruminal N require-

Table 4. Input descriptions of animals, management, and environment for optimization evaluations.[1]

	Lactating cows			Dry cows		Heifers	
	High	Medium	Low	Far-off	Close-up	Open	Bred
Lactation #	2.6	2.6	3	. . .	. . .	. . .	. . .
Age, mo	42	42	53	53	42	11	19
SBW, kg	600	620	636	636	636	270	409
Days pregnant	0	112	150	240	270	. . .	165
DIM	112	210	310	360	360	. . .	. . .
BCS	2.5	2.5	3	3.5	3.4	3	3
Milk, kg/day	43	30	22	. . .	. . .	. . .	. . .
Milk fat, %	3.5	3.5	3.8	. . .	. . .	. . .	. . .
Milk CP, %	3	3.2	3.5	. . .	. . .	. . .	. . .

[1] Breed: Holstein, calving interval: 13 mo, calving birth weight: 43 kg, age at first calving: 24 mo, mature weight: 660 kg, wind speed: 1.6 km/h, temperature: 7.2°C, relative humidity: 50%, hair depth: 0.64 cm, hide: thin, with night cooling, hair coat: 2 (mud on lower body and sides), mud depth: 2.5 cm. The physical activity equivalent to a tie-stall barn housing was used.

Table 5. Composition of feeds used to evaluate the optimizations.[1]

Composition[2]	AS	CS	GH	HMC	WCS	SH	SBM	SP	Urea	LS	CaS	DCP
$/t as-fed	30	22	80	150	160	130	270	290	240	100	140	380
Concentrate	0	40	0	100	100	100	100	100	100	100	100	100
Forage	100	60	100	0	0	0	0	0	0	0	0	0
DM, %AF	29	30	89	72	92	91	90	90	99	100	97	97
NDF, %DM	53	49.2	59.2	8.2	55.8	47.7	6	16.6	0	0	0	0
Lignin, %NDF	13	6.5	7.7	2.2	15	3	2.5	1.2	0	0	0	0
CP, %DM	18	8	12.8	9.4	22.7	14	54.5	49.6	281	0	0	0
SolP, %CP	55.7	47	25	19	20	24	18.5	10.7	100	0	0	0
NPN, %SolP	100	100	96	100	2.5	72	55	65	100	0	0	0
NDFIP, %CP	16	13	31	15.9	6	20	5	10	0	0	0	0
ADFIP, %CP	7.2	8.3	5.7	5.3	6	14	2	1.7	0	0	0	0
Starch, %NFC	45	100	44	100	90	90	90	90	0	0	0	0
Fat, %DM	4	4.5	2.9	4.3	17.5	2.9	1.1	7.6	0	0	0	0
Ash, %DM	8.5	3.9	8.5	1.6	5	5	6.7	6.3	0	100	100	100
eNDF, %NDF	82	81	98	5	100	2	23	30	0	0	0	0
CHO A	10	10	250	50	300	350	300	300	0	0	0	0
CHO B1	25	30	30	30	25	40	25	25	0	0	0	0
CHO B2	5.5	6	3	6	6	8	6	7	0	0	0	0
Protein B1	150	300	135	135	175	150	230	230	0	0	0	0
Protein B2	11	15	11	10	8	12	11	4	0	0	0	0
Protein B3	1.75	0.25	0.09	0.15	0.25	0.15	0.2	0.3	0	0	0	0
Calcium	0.7	0.3	0.3	0.4	0.2	0.5	0.3	0.3	0	34	23.3	22
Phosphorus	0.3	0.3	0.2	0.3	0.6	0.2	0.7	0.7	0	0	0	19.3

[1] AS = Alfalfa silage, CS = corn silage, GH = grass hays, HMC = high moisture corn, WCS = whole cottonseed, SH = soy hulls, SBM = soybean meal, SP = Soy Plus, LS = limestone, CaS = calcium sulfate, and DCP = dicalcium phosphate.

[2] Intestinal digestibilities were 100, 75, 20, and 0% for carbohydrate fractions A, B1, B2, and C, respectively; 100, 100, 100, 80, and 0% for protein fractions A, B1, B2, B3, and C, respectively; and 95% and 50% for fat and ash, respectively. HMC has 85% of CHO B1 intestinal digestibility.

Table 6. Evaluation of the original diets (kg/d, DM) fed to each group used as a starting point for the optimization.

	Lactating cows			Dry cows		Heifers	
Feeds (kg, DM)	High	Medium	Low	Far-off	Close-up	Open	Bred
Alfalfa silage	4	3.4	3.1	2.9	1.2	1.1	1
Corn silage	6.1	5.2	4.8	4.1	3.3	3	4.5
Grass hay	. . .	. . .	. . .	3.6	1.9	0.9	2.5
High moisture corn	5.7	4.8	4.4	0.9	2.7	1.4	0.9
Whole cottonseed	1.8	1.7	1.4	. . .	0.2	. . .	. . .
Soy hulls	0.9	0.8	0.7	. . .	. . .	. . .	. . .
Soybean meal	2.4	1.9	1.8	0.7	0.5	0.4	0.8
Soy Plus	1	0.9	0.8	. . .	0.3	. . .	. . .
Limstone	0.2	0.2	0.2	. . .	0.1	. . .	. . .
Calcium sulfate	0.1	0.1	0.1	0.1	. . .	. . .	. . .
Dicalcium phosphate	0.1	0.1	0.1	0.1	. . .	. . .	. . .
Actual DMI, kg/d	22.3	19.1	17.4	12.4	10.2	6.8	9.7
ME[1] balance, Mcal/d	1	5	7	6	1	0	0
ME balance, % required	101	111	117	127	103	100	100
MP[2] balance, g/d	58	203	292	329	6	40	182
MP balance, % required	102	110	117	141	101	106	123
Rumen N balance, %	127	126	126	122	164	101	110
ME allowable,[3] kg/d	44	35	28	. . .	. . .	1.25	1.38
Cost[4]	9.13	9.65	10.83	1.52	1.51	0.73	0.87

[1] Metabolizable energy.

[2] Metabolizable protein.

[3] Metabolizable energy allowable milk for lactating cows and average daily gain for heifers. Observed milk production for high, medium, and low groups is 43, 30, and 22 kg, respectively.

[4] The unit for lactating group is $/100 kg of milk, for dry group it is $/day, and for heifers it is $/kg of gain.

Table 7. Results of the diet optimization with the Cornell Net Carbohydrate and Protein System Version 4.0 for lactating cows.

	Group		
Feeds (kg, DM)	High	Medium	Low
Corn silage	15.3	8.76	3.95
Grass hay	. . .	3.64	6.10
High moisture corn	2.98	. . .	. . .
Soy hulls	0.21	4.64	6.17
Soybean meal	1.32	. . .	. . .
Soy Plus	2.04	1.69	0.91
Urea	0.09	0.03	. . .
Limstone	0.28	0.21	0.15
Dicalcium phosphate	0.12	0.13	0.12
DMI	22.3	19.1	17.4
ME[1] balance, Mcal/d	0	0	0
ME balance, % required	100	100	100
MP[2] balance, g/d	0	0	0
MP balance, % required	100	100	100
Rumen N balance, g/d	–5	0	–4
Rumen N balance, %	98	100	99
Milk predicted,[3] kg/d	42.9	29.7	22
Cost, $/100 kg of milk	7.47	8.36	10.48
Number of iterations	5	5	7

[1] Metabolizable energy.

[2] Metabolizable protein.

[3] Observed milk production for high, medium, and low groups is 43, 30, and 22 kg, respectively.

ment in lactating dairy cows, which is in agreement with NRC (19) based on a diet of 12 to 13% CP. Similar ranges in values for recycled N were reported by Al-Dehneh et al. (1) (19.1 and 7.4% for high-grain and high-forage diets, respectively). A maximum limit of 150% for eNDF was used to restrict the intake of eNDF to approximately 1% of BW.

Table 8. Results of the diet optimization with the Cornell Net Carbohydrate and Protein System Version 4.0 for dry cows.

	Group	
Feeds (kg, DM)	Far-off	Close-up
Corn silage	3.38	6.51
Grass hay	8.94	2.82
Soy Plus	. . .	0.76
Urea	0.02	0.04
Limestone	0.02	0.05
Dicalcium phosphate	0.03	0.02
DMI	12.4	10.2
ME[1] balance, Mcal/d	3	1
ME balance, % required	112	102
MP[2] balance, g/d	230	–3
MP balance, % required	127	100
Rumen N balance, g/d	19	3
Rumen N balance, %	109	102
Cost, $/d	1.18	1.10
Number of iterations	4	4

[1] Metabolizable energy.

[2] Metabolizable protein.

Along with an improved balance of ME and MP, the diet costs were lower for the optimized diets than for the nonoptimized diets (Table 7).

Dry Cow Groups

The results of the optimization for dry cows are shown in Table 8. This group required an average of four iterations to balance the diets. In the diets studied in this paper, the minimum constraints required were 100% for MP and 100% for ME. The maximum constraints were set at 130% for MP, and 120% for ME.

The minimum and maximum constraints set to optimize the diets for DMI were 100% of the actual DMI and the constraints for ruminal N balance ranged from 80 to 90% (Table 10), suggesting that the recycled N accounted for 10 to 20% of ruminal N requirements, which was similar to the lactating cow groups.

As with lactating cows, the optimized diets were less expensive than the nonoptimized diets and had a better nutrient balance. The cost ($/d) of the optimized diet for the far-off group was 1.18 versus 1.52 for the nonoptimized diet while for the close-up group it was 1.10 versus 1.51, respectively, for optimized and nonoptimized (Table 8).

Growing Heifer Groups

The number of iterations required to balance the diets to the ME requirement for the target average daily gain was seven for open heifers and five for bred heifers

Table 9. Results of the diet optimization with the Cornell Net Carbohydrate and Protein System Version 4.0 for replacement heifers.[1]

	Group	
Feeds (kg, DM)	Open	Bred
Corn silage	3.44	6.05
Grass hay	2.96	2.75
Whole cottonseed	0.36	0.79
Urea	0.01	0.04
Limestone	0.03	0.04
Dicalcium phosphate	0.01	0.02
DMI	6.8	9.7
Target ADG[2], kg/d	0.81	1.22
ME[3] allowable ADG[2], kg/d	0.85	1.29
MP[4] balance, g/d	23	51
MP balance, % required	104	106
Rumen N balance, g/d	5	12
Rumen N balance, % required	104	107
Cost, $/kg of ADG	0.76	0.73
Number of iterations	7	5

[1] Optimized to meet target weight gain.

[2] For bred heifers, average daily gain (ADG) includes conceptus requirement for growth (0.22 kg/d).

[3] Metabolizable enegy.

[4] Metabolizable protein.

Table 10. Constraints used as % of required for diet optimization for each group.[1]

	Lactating cows			Dry cows		Heifers	
Constraint	High	Medium	Low	Far-off	Close-up	Open	Bred
DMI							
Minimum	100	100	100	100	100	100	100
Maximum	100	100	100	100	100	100	100
ME[2] balance							
Minimum	100	100	100	100	100	100	100
Maximum	100	100	100	120	120	100	100
MP[3] balance							
Minimum	100	100	100	100	100	100	100
Maximum	100	100	100	130	130	110	110
Effective NDF[2]							
Minimum	100	100	100	100	100	100	100
Maximum	150	150	150	1000	1000	1000	1000
Ruminal N balance							
Minimum	80	80	75	80	80	75	80
Maximum	85	90	90	90	90	80	95

[1] Minimum and maximum for ruminal optimum peptide balances are 100 and 1000% (unlimited), and for calcium and phosphorus are 100 and 110%, respectively, for all groups.

[2] Metabolizable energy.

[3] Metabolizable protein.

[4] There is no maximum limit of eNDF for dry cows and replacement heifers.

(Table 9). The minimum and maximum constraints to optimize the diets for MP were 100 and 110%, respectively, for both groups. The constraints for ruminal N balance were 75 to 80% for the open heifers and 80 to 95% for bred heifers (Table 10). Thus, for the open heifer group, the recycled nitrogen accounted for 20 to 30% of the ruminal N balance. For the bred heifer group, the ruminal N balance was 108%, indicating a surplus of ruminal N. The maximum constraint for MP requirements was 110% in both groups in order to get a feasible solution. This was caused by the combination of microbial yield and feed undegraded intake protein from the basal feeds in their diet exceeding their MP requirements.

To optimize for the predicted target gain, we slightly modified the original composition of the diets to obtain an ME allowable gain similar to the predicted target daily gain. The optimized diets met the requirements for the target average daily gain at a cost lower than the nonoptimized diets for the bred heifers. The cost per day was lower in the optimized diet ($0.73) than the nonoptimized diet ($0.87) for the bred heifers.

APPLICATION

The procedure presented in this paper shows that linear programming can be used to optimize diets within the structure of the CNCPS. Consequently, this approach can be used to optimize dairy and beef cattle diets to meet their nutrient requirements while reducing the cost and minimizing nutrient excretion.

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