Least-Cost Ration Formulations for Holstein Dairy Heifers By Using Linear and Stochastic Programming

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ABSTRACT

Four mathematical programming models were developed to formulate rations for large breed replacement dairy heifers in each of 11 different weight classes from 50 to 550 kg and daily growth rates of 600, 700, and 800 g, with the objective of achieving a final calving weight of 600 kg. First, a base linear programming model was developed; then, to account for variability in the crude protein content of ration ingredients three other methods were used: right-hand side adjustment, incorporation of a safety margin, and stochastic programming. The average daily cost to calving, given a daily gain of 600, 700, and 800 g, was $0.62, $0.64, and $0.68, respectively. The total feed cost to 600 kg was $89.87 more for a growth rate of 600 over 800 g/d. The stochastic programming model performed better, on the bases of cost and protein-feeding, than did the right-hand side adjustment or the safety margin methods. The stochastic programming model over-adjusted crude protein by 5% and cost an average of 3.5% more than the linear programming solution for a dairy heifer growing at 800 g/d with a desired probability of 80% of crude protein intake achieving the NRC minimum. The other two methods over-adjusted crude protein by 10 and 13% and cost an extra 5.5 and 7.6%, respectively, for the right-hand side adjustment and the safety margin methods.

(**Key words:** mathematical programming, heifers, least-cost, economics)

Abbreviation key: **LP** = linear programming, **SM** = safety margin, **RS** = right-hand side, **SP** = stochastic programming.

INTRODUCTION

Raising dairy heifers to the age at which they enter the milking herd is one of the principal costs for dairy producers in maintaining a productive and profitable dairy herd. A major proportion of the expense of raising heifers, over 50%, is feed costs (3). A further problem of heifer raising is ensuring a heifer reaches the ideal calving weight of approximately 600 kg for Holsteins at approximately 23 to 24 mo of age (6, 14). Achieving the target weight requires careful nutritional management to ensure the feed ration contains the correct concentrations of essential nutrients. However, this may be a costly problem for some producers.

To reduce the cost of feeding, a least-cost ration formulation could be used. However, the nutrient requirements of growing animals differ as they progress through various stages of prepuberty, puberty, breeding, and gestation (10). To account for these differences makes calculating a least-cost ration more complex. One method that can be used to derive least-cost rations is linear programming (**LP**), which assumes first, that all inputs into the ration are infinitely divisible; and, second, their nutrient content is known (12, 13). The first assumption is valid in ration formulation, as it is possible to use as much or as little of an ingredient as desired.

A difficulty that arises in formulating dairy heifer rations is that heifers are usually fed a diet made up principally of forages, such as hay and silage, within which the nutrient content of the diet can vary widely. The variation in nutrient content could have a negative impact on the growth rate of the animals, particularly if a producer considers the ration has sufficient nutrient content based on mean values. As a result, the variation in nutrients should be considered when formulating the ration for each class of heifers in the replacement herd. The second assumption of LP may be invalid in most cases, as variation does exist in many feed nutrients. Several methods can be used to reduce the effects of variability in nutrient content in ration formulation. One of these methods is to incorporate a safety margin into the ration formulation; another is to increase the concentrations of the nutrient in the ration formulation. These methods are referred to as the safety margin (**SM**) formulation and the right-hand side adjustment (**RS**) model (12, 13). Another potential method is lead feeding, which accounts for variability in the requirements of a group of animals, such as milking cow groups—for which production varies throughout the group due to different stages of lactation—by calculat-

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ing a factor that is based on the variability of production (16). The nutrient requirements of the group are then multiplied by this factor; hence, lead feeding is similar to a combination of the safety margin and right-hand side adjustment formulations. Another method is stochastic programming (**SP**), which explicitly accounts for variation in the ration ingredients through the mathematical structure of the problem.

When no variability in the nutrient content of the ingredients of a ration is assumed, the typical method of formulation is to use the mean value for the nutrient in question. When variability does exist, it is possible to determine the probability that the nutrient meets or exceeds the requirements specified in the ration. By ignoring variability, the probability that the nutrient concentration in the ration exceeds the desired level is only 50%, assuming a normally distributed nutrient content (12). With other formulations, such as SM or SP, the probability of exceeding the desired level can be increased beyond 50%.

The mean values for an individual farm can be determined by forage or feed analysis, or the farmer may choose to use book values, such as those presented in the NRC Nutrient Requirements for Dairy Cattle (10). However, neither of these sources provides information regarding the variation of nutrient content in these feeds. Individual farm data allows the farmer to use specific information regarding the nutrient content of the feeds and forages. A farmer can calculate the variance (S) of a set of samples of a feed or forage using the standard formula:

$$S = \sum_{i=1}^{n}(x_i - X)/(n - 1)$$

where x_i = nutrient content of sample i, X = mean of the nutrient in the set of samples, and n = number of samples in the set. For example, this method would require a producer to submit more than one sample of feed or forage for analysis. The samples sent for analysis should be representative of those used in a ration. That is, a set of samples should come from the top, middle, and bottom of the face of corn silage stored in a bunker silo. This type of sampling would capture the variation in nutrient content of the silage, because nutrient content does vary during silage harvesting and subsequent ensiling (17). The mean and variances of nutrients within ration ingredients will also vary across farms and seasons, hence regular sampling and variance analysis would ensure better ration formulation.

The primary objective of this study was to derive a least-cost feed ration for dairy heifers within a confinement feeding system at various stages of growth by the above-mentioned methods and by incorporating ration ingredients that are readily available in the northeast of the United States. The nutrient of interest in the SM, RS, and SP models was CP because this is one of the major determinants of growth rate; other nutrients could have been selected but would not have added significantly to the applicability of the study. A secondary objective was to consider the additional costs incurred by a producer who uses a slower growth rate for the replacement heifers for the dairy herd.

MATERIALS AND METHODS

An LP least-cost ration formulation, in algebraic terms, has the following form:

$$\text{minimize } T = \sum_{j=1}^{n} c_j x_j$$

$$\text{subject to } \sum_{j=1}^{n} a_{ij} x_j \leq (\geq, =)\ b_1$$

$$x_j \geq 0$$

where T = total cost of ration, c_j = cost of ingredient j, x_j = quantity of ingredient j in the ration, a_{ij} = the quantity of nutrient i in ingredient j, and b_i = the required amount of nutrient i in the ration; the equality or inequality of the constraint is determined by the nutrient of interest.

The type of constraint, greater than, less than, or equal to, will depend on the nutrient or the nutrient balance required in the ration.

To incorporate an SM into the LP formulation, the desired probability that a ration contains a certain level of a nutrient must be defined. The requested probability is the probability that the nutrient is at the concentration determined by the ration formulator. The requested probability exceeds the probability of the LP ration. The LP ration implies a 50% probability that the nutrient is at the NRC level. The safety margin (SM_{ij}) is calculated as the sum, over each ingredient j, of the amount of nutrient i in ingredient j, a_{ij}, less the standard deviation of the nutrient, σ_{ij}, multiplied by the requested probability level. The requested probability level is the value generated from the cumulative standard normal distribution for a specific z-value (15). For example, if the requested probability is 80% then z = 0.83, ($0 \leq z \leq 2.33$). Algebraically, the full model is specified as

$$\text{minimize } T = \sum_{j=1}^{n} c_j x_j$$

$$\text{subject to} \sum_{j=1}^{n}(a_{ij} - z\sigma_{ij})x_j \leq (\geq, =)\ b_i$$

$$x_j \geq 0$$

A second potential method of ration formulation that allows for uncertainty or variation in the inputs is to increase the RS of the constraint. In this method, an increase in the level of constraint, b_i, by a percentage, such as 5 or 10%, is made to ensure the nutrient is at or above the desired level. Hence, the specification of the RS model is similar to that of the LP, except the RS of the constraints is written as φb_i, where φ = desired increase in the constraint, and in this context $\varphi \geq 1$.

The final method that allows for uncertainty or variation in input levels is SP. The SP has a form similar to the LP model, except the constraint terms, b_i, take into account the variability in the nutrients in the formulation. The constraints now include the nonlinear variance of the nutrient in each ingredient and a desired probability level. Thus, the SP model has the following form:

$$\text{minimize } T = \sum_{j=1}^{n} c_j x_j$$

$$\text{subject to} \sum_{j=1}^{n}(a_{ij} - z\left(\sqrt{\sum_{j=1}^{n}\sigma_{ij}^2}\right))\ x_j \leq (\geq, =)\ b_i$$

$$x_j \geq 0$$

where $(\sigma_{ij}{}^2)$ = variance of nutrient i in ingredient j, and z is defined as before.

The difference between the SM and SP models is the way in which variance is treated. In the SM formulation, the function is a linear function of the mean and the standard deviation, whereas in the SP model the formulation is a nonlinear function of the mean and variance (12). A simple example can demonstrate the mathematical advantage of SP over the SM method of analysis. By assuming two ingredients contain only one nutrient, the variance of the nutrient in each ingredient is 9 and 16, respectively. The SM model estimates the variability of the nutrient as $\sqrt{9} + \sqrt{16} = 7$. By using the formulation of the SP model, the variability of the nutrient is $\sqrt{(9 + 16)} = 5$. Hence, the SM model increases the effects of nutrient variability (5, 12, 15).

With the above methods, rations were formulated for dairy replacement heifers that included the nutrient requirements for each weight class specified. Required DMI, CP, NE_m, and NE_g and the macro-minerals, calcium, phosphorus, magnesium, potassium, sodium, chlorine, and sulfur, were based on NRC (10) standards and were defined for each 50-kg weight class from 100 to 550 kg, for large-breed growing females. The nutrient requirements for dairy heifers less than 100 kg were derived from Davis and Drackley (4) because the NRC standards do not list all the energy and CP requirements for heifers of this weight and at differing growth rates. The mineral requirements for dairy heifers weighing less than 100 kg were based on NRC (10) standards. The NDF percentage of the ration was set at a minimum of 30% up to 300 kg then increased to 35% for the weight classes above 300 kg because the NRC requirements do not include the NDF requirements for each of the defined classes. The NDF content of the diet was interpolated from Adams et al. (1) because the NDF was based on age class rather than weight. It is assumed that the growth rates of the heifers in each weight class are consistent with those expected within the NRC specifications and that the producer regularly weighs heifers to ensure the weight gain targets are met. If this were not the case, then an additional variable would have to be added into the constraints to capture the variation in growth rates and nutrient requirements within each weight class.

The nutrient and mineral content of the feeds used in this study were derived from the summary analysis of the Northeast DHIA Forage Testing Laboratory for December 1995 (11). These data were used because the summary contained measures of means and variances for the nutrients and feeds of interest. Other forage analyses usually do not report variation because they are single samples, hence the use of the summary data. However, if the means and variances of forage and feed nutrient content are available for an individual farm these should be used. Twenty feeds were used in the study: four types of hay (legume, mostly legume, mostly grass, and grass), straw, five types of silage (legume, mostly legume, mostly grass, grass, and corn), five energy feeds (high moisture ear corn, high moisture shelled corn, dry shelled corn, hominy feed, and wheat middlings), and five protein feeds (canola meal, whole cottonseed, dry distillers grain, soybean meal, and heated soybean meal). These feeds are those typically available to dairy producers in Pennsylvania on a continuous basis throughout the production year. A summary of means of the nutrient and mineral contents of each feed is shown in Table 1. The standard deviation of the CP content of all ration ingredients is also presented in Table 1, the standard deviations of the other nutrients are available, but are excluded from this table due to the required space. Three mineral supplements were also included as potential ration ingredients; these were monosodium phosphate (25% S, 19.2% Na), Dyna-

Table 1. Nutrient content and variability of crude protein of potential ration ingredients, derived from the Northeast DHIA Forage Laboratory summary (10).

Nutrient	DM (%)	CP (%)[1]	NE$_M$[2] (Mcal/kg)	NE$_G$ (Mcal/kg)	NDF (%)[1]	Ca (%)[1]	P (%)[1]	Mg (%)[1]	K (%)[1]	Na (%)[1]	S (%)[1]	Cl (%)[1]
Ingredient												
Alfalfa hay	91.30	19.40 ± 2.80	1.32	0.79	41.20	1.46	0.25	0.29	2.58	0.12	0.26	0.83
Mostly legume hay	90.80	16.40 ± 3.10	1.21	0.66	51.30	1.14	0.25	0.26	2.26	0.03	0.18	0.59
Mostly grass hay	91.50	12.10 ± 3.30	1.15	0.60	60.50	0.75	0.23	0.23	1.93	0.03	0.15	0.68
Grass hay	92.10	10.60 ± 3.20	1.17	0.62	64.80	0.55	0.22	0.16	1.84	0.08	0.19	0.55
Straw	93.00	5.00 ± 2.00	0.97	0.40	72.20	0.35	0.11	0.14	1.38	0.12	0.17	0.25
Alfalfa silage	40.40	19.20 ± 2.90	1.23	0.68	45.00	1.26	0.30	0.25	2.64	0.05	0.23	0.59
Mostly legume silage	39.10	17.40 ± 3.20	1.21	0.66	48.70	1.14	0.29	0.25	2.54	0.03	0.22	0.60
Mostly grass silage	37.10	14.00 ± 3.50	1.17	0.62	56.20	0.87	0.28	0.23	2.29	0.03	0.20	0.73
Grass silage	36.30	13.20 ± 3.90	1.17	0.62	59.40	0.67	0.29	0.22	2.35	0.08	0.22	0.84
Corn silage	33.10	8.10 ± 1.10	1.50	0.97	46.00	0.25	0.21	0.18	1.01	0.01	0.09	0.30
High-moisture ear corn	66.80	8.40 ± 1.00	0.89	0.64	21.20	0.03	0.27	0.12	0.48	0.01	0.08	0.07
High-moisture shelled corn	71.40	9.10 ± 0.90	0.97	0.65	10.80	0.02	0.30	0.13	0.42	0.00	0.09	0.05
Hominy feed	88.50	10.50 ± 1.40	1.06	0.72	18.80	0.04	0.44	0.18	0.58	0.01	0.10	0.06
Wheat middlings	89.60	19.30 ± 5.30	0.89	0.59	38.20	0.13	0.98	0.42	1.15	0.05	0.25	0.00
Dry shelled corn	88.10	9.10 ± 1.30	2.24	1.55	9.50	0.04	0.30	0.12	0.42	0.02	0.10	0.08
Canola meal	90.40	37.10 ± 2.60	1.59	0.99	28.60	0.79	1.20	0.56	1.14	0.09	0.64	1.07
Cottonseed	89.30	24.70 ± 3.00	2.40	1.70	51.00	0.16	0.60	0.37	1.19	0.01	0.22	0.06
Dry distillers grain	90.10	29.50 ± 3.90	2.18	1.50	38.70	0.18	0.80	0.32	1.08	0.25	0.40	0.21
Soybean meal	89.60	52.90 ± 3.40	2.07	1.43	10.50	0.40	0.71	0.29	2.34	0.11	0.39	0.05
Heated soybeans	92.50	43.30 ± 2.50	2.07	1.43	21.90	0.29	0.64	0.27	2.14	0.01	0.37	0.00

[1] Percentages per kilogram of DM.

[2] NE$_M$ = Net energy maintenance; NE$_G$ = net energy gain.

mate (22% S, 11.6% Mg, 18.5% K), and dicalcium phosphate (21% Ca, 18% P).

Prices for the different hay types were the averages for each type from the Pennsylvania Hay Market Summary of April 5, 1999. Silage prices were the average prices for Pennsylvania for the same period (8). Prices for the protein and energy feeds and mineral supplements were derived from Feedstuffs magazine and were adjusted for transport costs for delivery to Pennsylvania (8).

Table 2. Prices of potential feed ingredients.

Ingredient	$/kg as-fed
Alfalfa hay	0.14
Mostly legume hay	0.11
Mostly grass hay	0.11
Grass hay	0.09
Straw	0.09
Alfalfa silage	0.07
Mostly legume silage	0.06
Mostly grass silage	0.05
Grass silage	0.05
Corn silage	0.03
High-moisture ear corn	0.06
High-moisture shelled corn	0.09
Dry shelled corn	0.09
Hominy feed	0.13
Wheat middlings	0.11
Canola meal	0.17
Cottonseed	0.20
Dry distillers grain	0.16
Soybean meal	0.19
Heated soybeans	0.25
Monosodium phosphate	0.51
Dynamate[1]	0.51
Dicalcium phosphate	0.51

[1] IMC-AGRICO Co., Bannockburn, IL.

Linear programming models were designed for the initial rations and the SM and RS models. A nonlinear programming model was designed for the SP because of the inclusion of the nonlinear variance term. The LP model was used as the base situation, and all other models were compared with this base measure. There were two RS models estimated, one for an increase of 5% and the other for an increase of 10% in the RS constraint for CP. An increase in the constraint level of 10% is approximately equal to a requested probability of 80%. The SM and SP models were estimated with requested probabilities of 60, 69, 80, 90, 95, and 99%. Here 69 rather 70% was chosen because this represents a measure of one standard deviation from the mean. There were 15 models estimated each with three submodels for the three different weight gain levels and 11 submodels for the different weight classes. In all there were 495 different rations formulated in this study.

Ideally this research could have examined a compensatory growth pattern as suggested in Hoffman and Funk (7), in which a dairy heifer grows at approximately 600 g/d until puberty, and then the growth rate is accelerated to 1032 g/d postpuberty to achieve a calving weight of approximately 600 kg. However, the full nutrient requirements to achieve this growth rate are

Table 3. Calculated cost of feed for each 50-kg weight class with a growth rate of 800 g/d for each method and a requested probability of 80%, except for the linear programming (LP) solution.

	Solution method[1]			
Weight range (kg)	LP ($)	SM ($)	RS ($)	SP ($)
50–100	11.07	11.65	11.68	11.61
100–150	18.42	19.63	19.46	19.25
150–200	23.65	25.25	25.03	24.60
200–250	27.82	29.89	29.52	28.89
250–300	30.07	31.91	31.41	31.01
300–350	35.86	38.70	37.64	36.99
350–400	41.39	44.97	43.74	42.88
400–450	47.69	51.82	50.40	49.29
450–500	54.54	59.27	57.64	56.25
500–550	62.17	67.56	65.70	63.99
550–600	70.79	76.92	74.81	72.73
Total feed costs	423.47	457.57	447.03	437.49

[1] SM = Safety margin, RS = right-hand side, and SP = stochastic programming.

not fully specified, and to maintain a consistent base of study, this rate of growth was not included in this study.

All models were estimated by using GAMS/MINOS (2), on a SUN/SPARC system. The GAMS/MINOS can be used to solve linear and stochastic models, because it is a nonlinear programming solver, thus the LP models and the SP model can be solved using a single solver.

RESULTS

Given the large number of potential results it is not possible to present all rations, thus a small number of resultant rations will be presented. It is possible to interpolate from these the formulations for other rations not reported. Table 3 presents the total costs for a ration to bring a dairy heifer through each weight class (from 50 to 550 kg) at a daily growth rate of 800 g/d for the LP model with the constraints that all nutrients at least achieve the NRC requirements. Table 3 also presents the results for the SM, RS, and SP models with a probability of 80% that the CP content of the ration be at least equal to the NRC requirements; all other constraints were as in the LP formulation. The total costs for feeding a dairy heifer growing at 800 g/d varied from $423 to $457. Figure 1 shows the calculated CP content of a daily ration for each of the corresponding models of Table 3. When the dip in CP content is between the weight classes of 200 to 250 kg and 250 to 300 kg it is due to the drop in NRC requirements for CP (10). The calculated CP levels varied across rations; the SM ration overfed CP by 9.5 to 14.4%, and the RS rations overfed CP by 10%, as expected, because of the formulation of the ration. The ration formulated by the SP overfed CP by 3.7 to 8.8%.

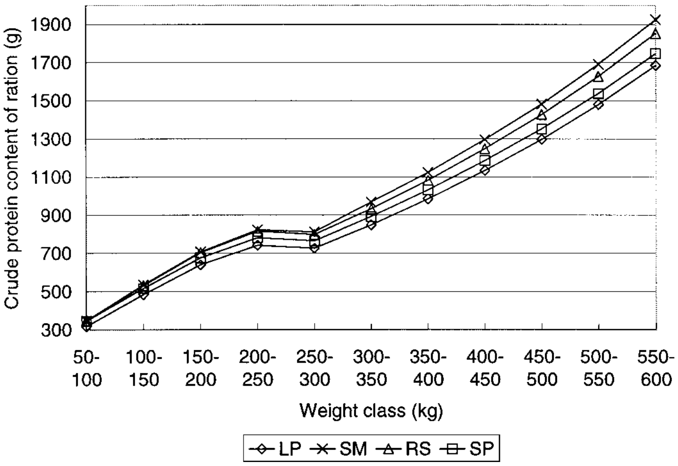

Figure 1. Calculated daily CP contents of rations formulated for growth rate of 800 g/d for each method and a requested probability of 80%, except for the linear programming (LP) solution. The LP solution levels for CP are equivalent to the NRC requirements. SM = safety margin, RS = right-hand side, and SP = stochastic programming.

The variation in ration CP content is due to the method used to calculate the actual ration CP content, not the adjusted CP content as determined by the mathematical formulation of the ration (12). The actual ration CP uses the ingredients, as determined by the respective formulation, and multiplies these by the CP content of the ingredient.

Figure 2 summarizes the daily feed costs for LP ration developed for the three different daily weight gains and the 11 weight classes. From this figure it is possible to see how the cost of feed varies as the animal gains weight. The daily costs of feed are lowest for the slowest

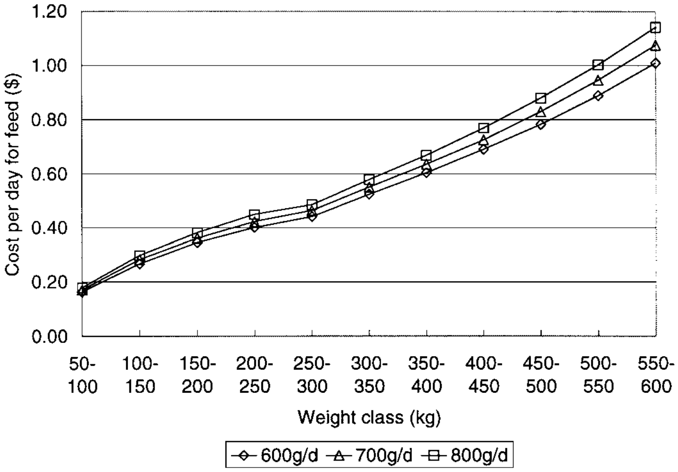

Figure 2. The cost per day for feed from the linear programming models for growth rates of 600, 700, and 800 g/d.

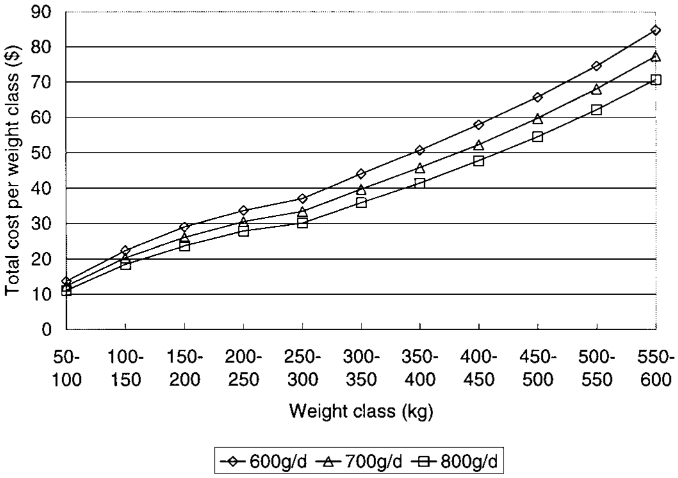

Figure 3. The total cost of feed per 50-kg weight class from the linear programming models for growth rates of 600, 700, and 800 g/d. At 600 g/d a heifer takes approximately 84 d to grow 50 kg. At 700 g/d a heifer takes approximately 72 d to grow 50 kg. At 800 g/d a heifer takes approximately 63 d to grow 50 kg.

growth rate and highest for the fastest growth rate. Figure 3 demonstrates the effects of aggregation on the total feed costs of dairy heifer rearing for the LP model. From this figure it can be seen that the total feed costs are in reverse position to those of the daily feed cost figure because of the longer period that heifers are in each weight class when growing at slower rates. Figure 4 presents the differences in total feed costs for each weight class on total cost for the respective growth rates (600, 700, and 800 g/d) and solution methods. In this figure it is possible to see that a dairy heifer growing at 600 g/d consistently costs more on a cost per weight class change than does a heifer growing at 800 g/d, irrespective of the way the ration is formulated. The principal reason is that the animals require a substantial amount of nutrients for maintenance, so the costs of maintaining an animal at a lower rate of weight gain results in a higher cost to rear that animal to the desired weight.

Ration ingredient percentages for each weight class and method of analysis are presented in Figure 5, where the percentages are on an as-fed basis. These figures provide comparisons between the rations formulated for the different weight classes at a constant growth rate of 800 g/d. Not represented in each figure is a small amount, between 5 and 80 g/d, of various combinations of the mineral supplements due to the relatively small fraction in the complete ration. The ration ingredients across each model are relatively similar, consisting principally of grass hay, mostly legume hay, and soybean meal with a slight amount of minerals fed to supplement the natural mineral content of ingredients in the rations. Dry shelled corn also included all rations for the 50- and 250-kg weight classes. The major difference across rations is the proportion of some ingredients, relative to other rations. Taking the 50-kg weight class,

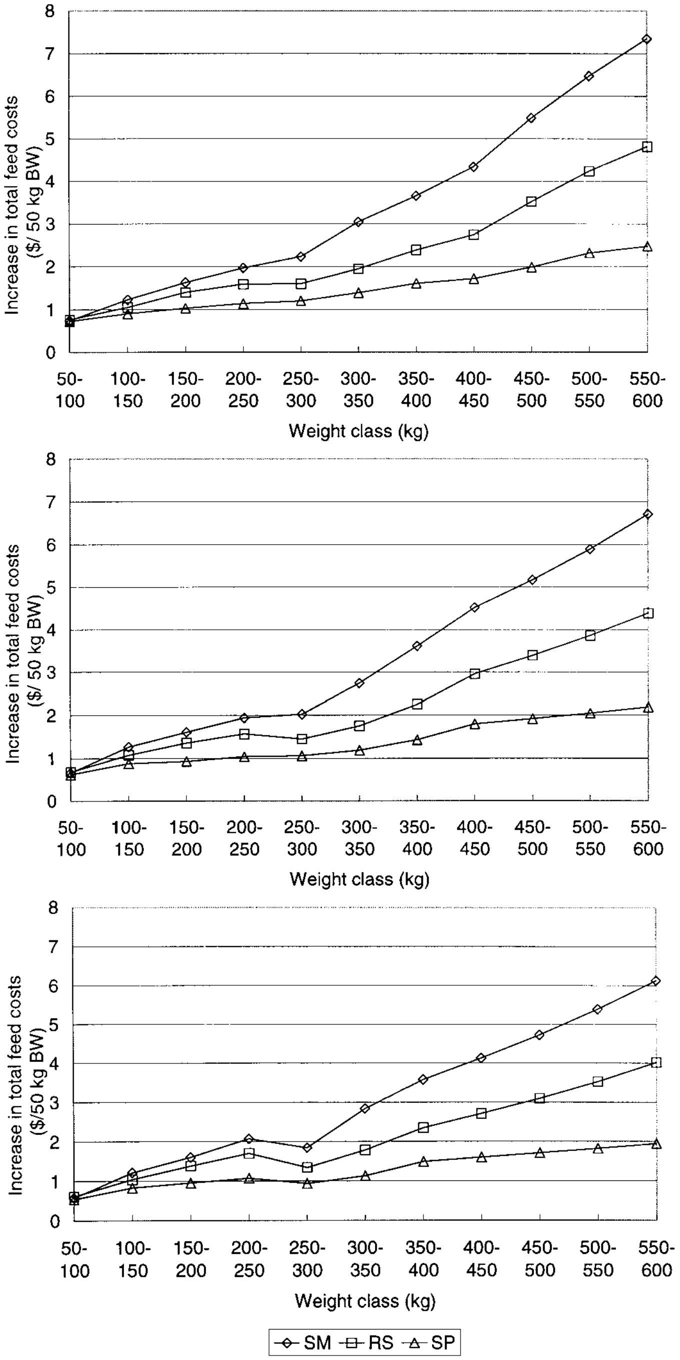

Figure 4. The increase in total feed costs per 50-kg weight class from the linear programming solutions to the safety margin (SM), right-hand side (RS), and stochastic program (SP) model solutions for a growth rate of 600, 700, and 800 g/d, respectively, and a requested probability of 80%.

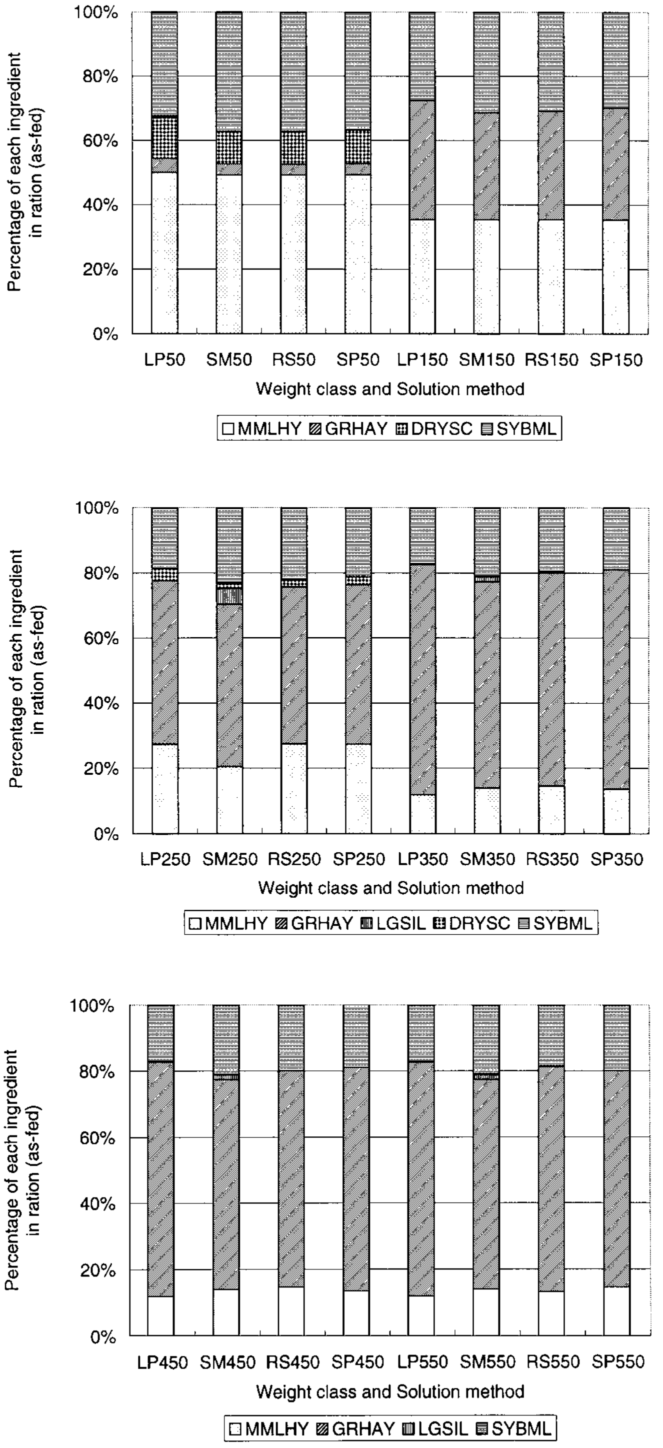

Figure 5. The percentage of ration ingredients in each ration for a heifer growing at 800 g/d in weight class 50 to 100 kg (represented by the 50 suffix on the abbreviations for each solution method), 150 to 200 kg (150), 250 to 300 kg (250), 350 to 400 kg (350), 450 to 500 kg (450), and 550 to 600 kg (550) LP = linear programming, SM = safety margin, RS = right-hand side, MMLHY = mixed mostly legume hay, GRHAY = grass hay, DRYSC = dry shelled corn, SYBML = soybean meal, and LGSIL = legume silage.

Table 4. The as-fed weight of each ration for alternative weight classes.

	Solution method[1]			
Weight range (kg)	LP (kg)	SM (kg)	RS (kg)	SP (kg)
50–100	1.29	1.33	1.33	1.33
150–200	2.91	3.05	3.03	2.99
250–300	4.01	4.23	4.09	4.07
350–400	5.82	6.16	6.02	5.95
450–500	7.67	8.12	7.93	7.81
550–600	9.95	10.54	10.12	10.29

[1] LP = linear programming, SM = safety margin, RS = right-hand side, and SP = stochastic programming.

for example, the proportions of mixed mostly legume and grass hays were relatively invariant. However, the proportions of dry shelled corn and soybean meal were varied across the rations. This variation in ingredient quantities is caused by the differing requirements for each ration because of the constraints imposed in each model for the CP content and how the variability of CP is handled across models. All other constraints are identical in all models. Even though the proportions of each ingredient are relatively consistent across models, differences occur in the total as-fed weight of each ration because of the ingredient variations across rations and ration contents. Table 4 summarizes the effect nutrient variability has on the as-fed weights of particular rations.

The ration ingredients also varied when the required probability increased from 50 to 99%. Selected stochastic program model results are presented in Figure 6. Because the required probability increased, the composition of a diet for a dairy heifer in a particular weight class changed. This response to changing the probability was due to the model selecting inputs that were less variable with respect to CP content at the expense of ingredients that had higher CP variability. The changes in ration formulation are particularly evident in the ration for a heifer in the 50-kg weight class. An increase in the desired probability from 50% to the upper limit of 99% caused the ration to change. The LP ration was a mixture of legume and grass hay, dry shelled corn, and soybean meal, whereas the high probability ration contained relatively lower proportions of legume hay and shelled corn, increased levels of soybean meal, and included legume silage.

In Figure 6, it is possible to see that as the desired probability increases the model formulation changes the content of ingredients. Ingredients that have lower relative variability are added, and ingredients that have relatively higher variability of CP content are removed, or the proportion of the ration ingredients is reduced. As dairy heifers approach the ideal weight, the ration formulation and ingredient proportions stabilize as exhibited in Figure 6. However, it is possible to see the effect of CP variability on ration formulation. As the proportion of grass hay declines because of a relatively high CP variability, the proportion increases for the relatively less variable mixed mostly legume hay. Finally, legume silage is introduced in the ration, because of the lower variability of CP in this ingredient.

Accompanying the increase in requested probability is an increase in the cost of each ration. Using the 50-kg weight class as an example, an increase in the requested probability from 50 to 80% causes a 5% rise in feed costs, from 50 to 90% leads to an 8% increase, and from 50 to 99% leads to a 15% increase in feed costs. A similar trend occurs in the 550-kg weight class. However, the percentage increases are approximately half that for the 50-kg weight class. Overall, the total feed costs increased in a linear trend from approximately 1 to 9.5% as the requested probability increased from 50 to 99%.

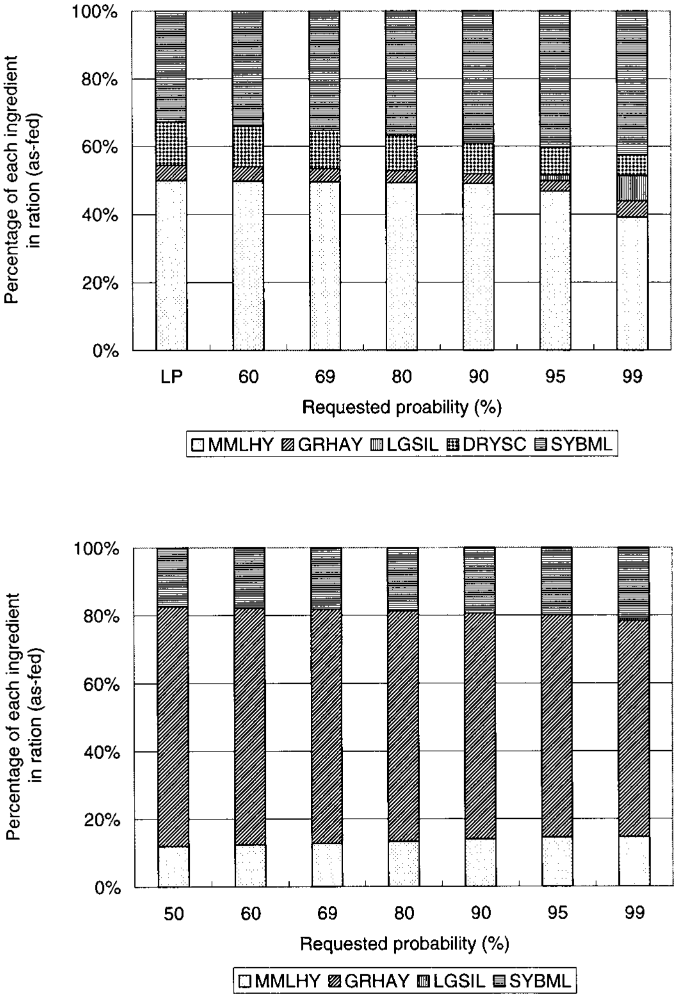

Figure 6. The percentage of ration ingredients in a ration for dairy heifers weighing 50 and 550 kg, respectively, growing at 800 g/d from the stochastic programming models with the associated probability levels. LP = linear programming, MMLHY = mixed mostly legume hay, GRHAY = grass hay, DRYSC = dry shelled corn, SYBML = soybean meal, and LGSIL = legume silage.

DISCUSSION

Formulation of a ration for growing Holstein heifers must take into account the changing nutritional requirements of these animals as they grow. Also, the ration should be the least-cost formulation to enable a dairy producer to grow replacements at the lowest cost, yet still have those heifers entering the milking herd at an ideal weight and age to improve the probability of longevity, high production, and profitability.

The costs of formulating a ration varied with the type of ration fed and the weight of the animals being fed. This expense also changed when the ration was formulated to account for variability in the CP content of the ingredients. Rations had the lowest cost when formulated to ignore the variability of the CP content of ingredients. This result was expected because the LP models simply selected the combination of ingredients that minimized cost and met the constraints imposed. Changing the constraints of the ration to account for the variability imposed additional costs, and the size of these additional costs depended on how the CP variability was incorporated into the ration. The two costliest methods were imposing an SM and increasing the required CP content of the ration. However, by changing the model formulation to directly account for the variance of the CP content in each ration ingredient as in the SP model, the cost of the ration increased but at a much lower level than the SM or RS formulations, for all rations formulated. Although a small sample of results is shown here, these results are typical for all rations developed. Choosing a different set of safety margins, RS adjustments and probability levels would yield similar results providing the set is consistent across probability levels (5).

The SP formulation allows for a much better control over the probability of achieving the desired overall nutrient content in a ration. The other two methods overcompensated for the CP content, and the rations derived in the study by the SM and RS had substantially higher levels of CP than specified by the NRC requirements.

Another aspect to consider when rearing replacement dairy heifers is the cost involved in taking a longer time to achieve the ideal 600-kg weight. The costs involved include the direct costs, such as feed and labor, and the opportunity or lost costs of not having a dairy heifer calve at 2 yr. At a daily growth rate of 600 g/d it takes approximately 30 mo for a heifer to grow to 600 kg; at 700 g/d, the time span is 26 mo, and at 800 g/d, this time is reduced to approximately 24 mo. For the lowest growth rate, the total feed cost is approximately $90 more than the cost of a heifer growing at 800 g/d. A dairy heifer growing at 800 g/d and calving at 24 mo has completed 6 mo of her lactation before the heifer growing at 600 g/d calves. In this case, the heifer costs the producer the additional feed and other costs, such as housing and labor, and the opportunity cost of 6 mo of milk production and milk revenue from that heifer. Research has shown that milk production and the productive life of a heifer is reduced by calving heifers at ages younger than 22 to 23 mo (6); however, production is not enhanced by calving cows at older ages (9).

CONCLUSIONS

The method in which nutrient variability is incorporated into a formulated ration affects the cost of the ration and the ingredient content of the ration. Stochastic programming minimized the ration cost and level of overfeeding of CP compared with adjusting the nutrient level through an SM or RS adjustment.

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