# Lactation Yields and Accuracies Computed from Test Day Yields and (Co)Variances by Best Prediction

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## ABSTRACT

Lactation records are calculated from data on milk, fat, and protein obtained from one or more milkings on several days during the lactation. The test interval method, which estimated missing daily milk yields by simple interpolation, was used for many years for standard monthly data but may not be as useful for the wider variety of test plans now being proposed. More accurate 305-d yields can be computed using best prediction, which has optimum properties if means and (co)variances are known and distribution is multivariate normal. The covariance of test day and 305-d yields is multiplied by the inverse of the test day (co)variance matrix, which is then multiplied by the test day deviation vector. This predicted 305-d deviation plus the mean 305-d yield equals the predicted 305-d yield. Similar algebraic methods are used to compute the correlation of true and estimated 305-d yields, which is needed to calculate lactation weights. Computation times were affordable but not trivial; they ranged from 0.001 to 1 s per lactation. Equations were modified to account for differing accuracies of data for partial days, means for multiple days, and data for unsupervised tests. Complete or incomplete lactations recorded with very different testing plans can be graphed and compared by best prediction.
(**Key words**: test day yield, test interval method, best prediction, lactation weights)

**Abbreviation key**: **DCR** = data collection rating, **LER** = labor efficient record.

## INTRODUCTION

Milk recording in the US is less uniform than in the past; several of the plans are familiar, but many more innovative plans exist as well. Until recently, monthly testing and sampling provided test day data that fit into fairly simple formulas that gave accurate and consistent lactation records. New test plans with varying test intervals, incomplete data on test day, reduced supervision, and electronic recording can provide less expensive lactation records with lower or even higher accuracy. A wide variety of plans can provide useful data for genetic evaluations, but data should be weighted according to accuracy and combined using improved formulas that adapt to data design.

Lactation totals are calculated by the test interval method in many nations. This method was invented in 1880 (8) and became the official method for calculating US lactation records in 1969 (15). Lactation totals are calculated from test day yields by “connecting the dots” using linear interpolation between data points as shown in Figure 1. Shook et al. (11) improved the test interval method by deriving a set of factors that better estimated daily yields before the first test, after the last test, and at peak yield. The amended test interval method performed well when applied to monthly testing and sampling but was not designed for the wider variety of plans now used.

Variables for record standards allowed the US dairy industry to describe differences between test plans more completely. In 1996, four new variables (numbers of supervised tests, unsupervised tests, supervised component samples, and unsupervised component samples) and three previous variables (DIM, test plan, and lactation number) were used to calculate lactation weights (13). Lower weights were assigned to records with unsupervised tests, missing component samples, or tests that were less frequent than once per month (13). Although variables for record standards are an improvement, formulas based on these variables are approximate rather than exact. Exact formulas for estimating lactation records and computing lactation weights depend on the intervals between test day observations for milk, fat, and protein and the correlations among them.

Random regression (9) allows the fitting of lactation curves to individual lactations. Because curve parameters are treated as random variables, reasonable estimates are obtained even with few data points. Unusual data patterns are regressed toward the average lactation shape observed within a herd. Lactation

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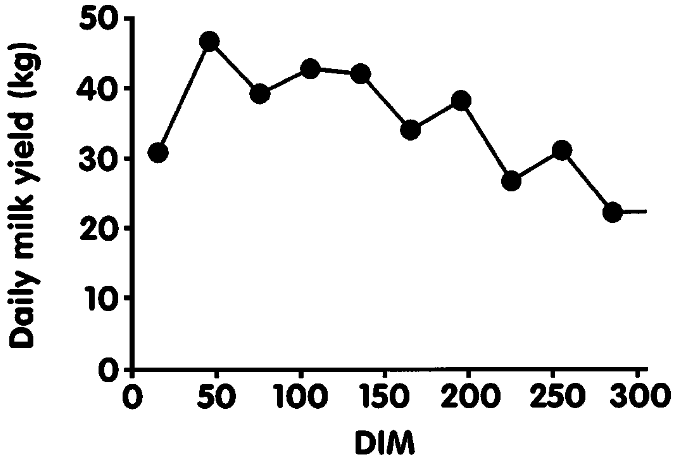


Figure 1. Example lactation plotted by the unadjusted test interval method (14) where ● = supervised milk weight.

curves for future cows in a herd or future daughters of a sire are predicted from these parameter estimates.

Best prediction considers just one lactation of one cow and uses those observed yields to predict daily yields that were not observed. If the test day yields are multivariate normal, all of the information about curve shape is contained in the means of the daily yields and the covariances among the deviations (12). Thus, best prediction is simple and has maximum precision if means and covariances are known. With random regression, some covariances among deviations are ignored (10). Because a few parameters can provide a close fit to the many correlations present in a 305 × 305 matrix (5), best prediction and random regression could produce similar answers using different algebraic methods.

Genetic evaluations can include test day data either directly as test day models (6, 16) or indirectly after lactation yields are estimated. Test day models can describe biology and define management groups more precisely but result in large sets of equations. Simpler sets of BLUP equations were used for national evaluations in Australia (2, 4) and New Zealand (1) and by other researchers (7, 10) to adjust for environmental factors affecting each test day before lactation totals are calculated.

An even simpler approach is to apply best prediction to test day yields one lactation at a time. For each day of lactation, the most probable producing ability is computed using selection index methodology. The lactation total is then the sum of the daily yields without adjustment for test day environment. The single equation used in this selection index approach is less accurate than the simultaneous equations of BLUP but more accurate than the test interval method. Best prediction allows data from different cows and from widely differing test plans to be graphed, compared, and weighted fairly simply.

Objectives of this research were 1) to use best prediction to compute and to plot lactation yields; 2) to account for unusual test intervals, data averaged over multiple days, estimates from partial days, missing traits, lack of supervision, and mixtures of test plans; and 3) to derive the accuracy and an appropriate lactation weight for any particular record.

## MATERIALS AND METHODS

Correlations between any two test day yields were estimated by Norman et al. (5). Sampling variances and memory were reduced by fitting smooth functions including linear and quadratic regressions on the difference in DIM, mean DIM, and an interaction. From these test day correlations, the correlations or squared correlations of estimated and true 305-d yield can be computed. These correlations are higher than originally reported by Norman et al. (5) because one herd with erroneous data was later detected and deleted.

McDaniel (3) summarized earlier research on the accuracy of different test plans. These studies applied the test interval method to different subsets of test day data and then examined properties of the resulting records. Variances of the difference $\left(\sigma_e^2\right)$ between 305-d yields obtained from daily yields and from several other plans were available and were compared with variances of true yield within management groups $\left(\sigma_y^2\right)$. The value for $\sigma_y^2$ was not reported by McDaniel (3) but instead was approximated using the mean standard deviation for Holsteins in the early 1960s (17). Squared correlations of estimated and true 305-d yields were then computed as $\sigma_y^2/\left(\sigma_y^2 + \sigma_e^2\right)$.

The accuracy of a hypothetical test plan may be determined by applying algebra to the test day (co)variance matrix instead of creating or waiting for a large file of data from such plans. Different recording procedures over different intervals can be compared easily for their abilities to predict true 305-d records or records of other length. Even more generally, the accuracy of each individual record can be computed from the test day observations in that lactation.

### Theory

Test day yields with multivariate normal distribution, known variances, and known means can be combined into a 305-d yield by best prediction. Unless data are simulated, estimated means and variances must be used in place of true means and variances. Then, selection index procedures can be used to predict the many missing daily yields from their covariances with the few measured yields. The lactation yield is the sum of the daily predicted or measured yields.

Lactation records for a particular trait can be obtained by single-trait prediction from test day data for just that trait or by multiple-trait prediction. Single-trait lactation records derived from one trait are simpler to model, but multiple-trait records are more accurate, especially when some component samples are missing (10). Genetic evaluations may soon predict genetic effects directly from test day data without first creating a 305-d record (16); however, lactation yields will continue to be widely used in the dairy industry.

Using matrix algebra, a 305-d yield ($\hat{y}$) equals the mean 305-d yield ($\mu$) plus the covariance between lactation and observed test day yields ($\mathbf{c}$) multiplied by the inverse of the variance for observed test days ($\mathbf{V}^{-1}$) multiplied by the observed test day deviations ($\mathbf{t}$):

$$\hat{y} = \mu + \mathbf{c}'\mathbf{V}^{-1}\mathbf{t}.$$

Dimensions of $\mathbf{t}$ and $\mathbf{V}$ are usually <30 with multitrait prediction and <10 with single-trait prediction of 305-d yields for milk, fat, or protein. Instead of predicting each daily yield and then summing the daily predictions, computations were reduced by summing first so that elements of $\mathbf{c}$ contained covariances for lactation rather than daily yields. Squared correlations ($r^2$) of predicted and true 305-d yields are $\mathbf{c}'\mathbf{V}^{-1}\mathbf{c}/\mathrm{Var}(y)$, and lactation weights (w) for an animal model with repeated records are $(1 - \mathrm{rpt})/[(1/r^2) - \mathrm{rpt}]$, where rpt is repeatability of lactation yield (14).

## Tests for Partial Days

The expected values of tests for partial and full days should be equal if the a.m. and p.m. yields are adjusted to a 24-h basis using appropriate factors for number of milkings and milking interval. The variance of a test for a partial day should be larger than the variance of a test for a full day because an additional measurement error is introduced when the 24-h yield is estimated. Covariances among tests for partial and full days are not affected because measurement errors for different days are assumed to be independent. Only the diagonals (variances) for tests for partial days are increased.

To determine the variance in measurement error, squared correlations obtained by T. K. Meinert (USDA, 1995, unpublished data) were compared with squared correlations that had been obtained theoretically. For theoretical correlations, the same proportional increase in variance was assumed for milk, fat, and protein and across all DIM. Alternatively, actual increases in variance for each trait or stage of lactation might be justified. For this study, the theoretical increase was set equal to average actual increase.

The milking frequency for a herd also affects measurement error. Plans with a.m.-p.m. tests measure about one-half of the daily yield of cows with twice daily milking but either one- or two-thirds of the yield if cows are milked three times daily. Longer or shorter milking intervals probably also affect the accuracy of the test but exact milking times may not be reported. Measurement error $\left(\sigma_m^2\right)$ was assumed to decrease as the proportion of milk tested increased according to the function $\sigma_m^2 = a\sigma_y^2[(\text{number of milkings/number of tests}) - 1]$ where the constant a was determined from data from herds milked twice daily. Data for herds milked three times daily or more frequently were not readily available, but the percentage of daily milk tested determined $\sigma_m^2$. Specifically, the $\sigma_m^2$ for a herd milked three times and tested at two milkings would be half as large as $\sigma_m^2$ for herds milked twice and tested once; $\sigma_m^2$ for a herd milked three times and tested once would be twice as large. If milking times are recorded, the formula $\sigma_m^2 = a\sigma_y^2[(24\ \text{h/h of milk recorded}) - 1]$ may estimate $\sigma_m^2$ more precisely. Both formulas should be verified.

Errors on the same day for milk, fat, and protein are expected to covary for a.m.-p.m. testing; the covariance would be determined by the number of milkings at which neither of the two traits is measured. For example, if protein is tested at one of three milkings but milk is recorded at two of three milkings, the third milking at which neither trait is measured causes a common measurement error. The second milking would not contribute to error covariance because milk is assumed to be measured without error at that milking. More generally, the covariance of traits was multiplied by a[(number of milkings/milkings tested for either trait) – 1] to account for a.m.-p.m. measurements of different traits on the same day.

## Means for Multiple Days

With labor efficient records (**LER**), measurement error for milk yield is reduced because consecutive daily weights are averaged. Component percentages, if measured, are multiplied by the mean for milk yield to determine fat and protein yields. Variances, covariances, and expected values of LER tests can be generated from corresponding parameters of the daily milk yields included in the mean. Each component yield was assumed to be a simple daily yield because covariances among yields and percentages were not available.

Means of electronically stored weights may be more accurate than monthly testing by either the owner or a supervisor. Daily fluctuations in milk yield are minimized when yield is reported as the mean of yields for several days. The mean yield is then assumed to be the daily yield on either the last day (sample day) or perhaps the center day of the period over which the mean was taken. The fat and protein percentages on test day then are multiplied by the test day milk yield or mean milk yield to obtain component yields. If test day milk is used in this calculation, reported yields and percentages are inconsistent; for example, a reported protein percentage of 3.0 and protein yield of 1.5 kg would agree with the 50 kg of milk produced on test day but not with the mean milk yield actually reported. If mean milk yield is used, component yields do not reflect the correlation that may occur between sample day milk yield and component percentages; for example, if milk yield were higher on sample day and fat test were lower, the cow would be credited with lower mean milk yield and the low fat test.

Currently, percentages for test day components are multiplied by means for 5 or 7 d of milk yield. In future LER tests, lactation yields of protein and fat could be more accurate if the yields for milk, fat, and protein on test day were separated from the mean for milk yields from previous days. When component samples are taken, two data segments could be constructed: one for the data on test day and one for a mean of up to 30 d of milk weights stored prior to test day. Means longer than 30 d are not recommended because proposed test day models use monthly time units to compare cows and to estimate persistency. When component samples are not taken, only one data segment would be needed for the mean for milk yield.

## Unsupervised Data

Data may be recorded by an on-farm computer, a supervisor, or the herd owner. Traditionally, trained supervisors worked full time to record accurate data. With unsupervised data, more errors may occur because herd owners have less training and may have financial incentives to report higher or lower yields than were actually obtained. For example, a herd owner could exaggerate and report that his best cow produced a world record or report that the daughters of one sire always produced more than the daughters of other sires. For this reason, lower weights may be justified for owner-sampler data than for supervised data. Owner-sampler data were excluded from USDA-DHIA evaluations prior to 1997 and thus received no weight.

Measurement errors from an owner might be random and independent from month to month or might be systematic and affect the entire lactation. If errors were random, larger variances would be expected but without change in the covariance between test days. Then, the same mathematical procedures as for a.m.-p.m. tests could be employed. If errors were systematic, the owner's error could be modeled as a permanent environmental effect across the lactation. Equivalently, the permanent environmental effect could be included in the phenotypic variance to reduce computation. Then, covariances and variances would both increase by the variance of the measurement error for test day. Because errors may be proportional to phenotypic variance, a fixed constant was added to the correlation (rather than the covariance) among any two test days, both measured by the owner.

The size of the fixed constant was determined by committee rather than by actual estimation. The desired outcome was to weight owner-sampler records by 75% as compared with supervised records. Because errors by the herd owners are treated as correlated, any supervised tests in the lactation then receive somewhat more weight because they are treated as truly independent observations.

Data that are obtained from on-farm computers may be coded as supervised or unsupervised. Further study is needed to determine the true range of errors that are likely or possible with automated recording systems. All data must meet the standards of the National DHIA program of quality certification. Meters, laboratory methods, and data processing are monitored so that measurements are accurate and comparable whether milk weights and samples are taken by a supervisor, owner, or automated device.

## Herd Means

Test day means are assumed to be known in theory but, in practice, can be generated from the lactation mean for each herd. The yield for each day as a fraction of total 305-d yield can be computed from one or more standard lactation curves. These curves could differ by breed, parity, or production level of the herd.

A mean of 305-d records adjusted for age, parity, season, and days open is then assumed to be the true mean for the herd-year or management group. Test day data for the cow of interest should be adjusted for these same effects, or the standardized herd mean could be unadjusted instead to match environmental conditions of the particular cow. This second option is recommended for plotting so that the raw data for the cow and the estimated lactation curve are both expressed in the observed units.

### Heritabilities

Only phenotypic means and variances were needed to derive best prediction, but heritabilities and weighting factors may also be affected by the choice of scale and methodology. Previously, monthly testing was the standard, and 305-d records with 10 monthly tests received a weight of 100%. Higher accuracy of LER testing now suggests that daily testing should become the standard simply to prevent weights of >100%. Thus, only a record with 305 daily tests would receive a weight of 100%. Heritability of the standard record would rise accordingly. The choice of which test plan is used as the standard does not affect the actual regression of genotype on phenotype ($b_{g.p}$) for any plan. With a repeatability of 55%, the two methods of expressing $b_{g.p}$ for a particular record are $b_{g.p}$ = old heritability/[0.55 + (0.45/old weight)] and $b_{g.p}$ = new heritability/[0.55 + (0.45/new weight)]. Suppose that the previous heritability was 0.25 and the previous weight for monthly testing was 1.0. With a new weight of 0.94 for monthly testing, the new heritability must be 0.26 if $b_{g.p}$ is to remain constant.

### Computation

A Fortran program was developed to estimate yields by best prediction and to compute correlations of true and estimated yields. Performance of the program was tested on two IBM computers (model 9370 mainframe and model RS/6000 workstation; IBM, Armonk, NY). Actual times of completion were recorded while computers were otherwise idle. Memory was greatly reduced by generating covariances from a function as needed (5) instead of storing the 365 × 365 or 1095 × 1095 covariance matrices used in single-trait or multiple-trait predictions, respectively.

Test days after d 305 can help in predicting 305-d yields (10). The first test after 305 d but limited to 365 d was accepted in the data vector. For lactations <305 d, the incomplete record can also be summed by best prediction of all daily yields previous to the current test day. The same algebraic formula is used for partial, completed, and projected records.

A measure of accuracy, termed the data collection rating (**DCR**), was defined for use by farmers and breeders. The DCR is the squared correlation of predicted and true lactation yields multiplied by 100 and divided by the squared correlation for a standard, supervised plan with 10 monthly tests. With this definition, a rating of 100 is reserved for standard monthly testing instead of for daily testing. Reporting DCR for lactation records is analogous to reporting reliabilities of breeding values.

## RESULTS AND DISCUSSION

Multiple-trait prediction for a lactation with 10 test days required inversion of a 30 × 30 matrix plus a few other algebraic steps. The time required was almost 1 s per lactation on a mainframe computer and 0.02 s on a workstation. Single-trait prediction required inversion of three 10 × 10 matrices plus other steps. Total time was much less: only 0.05 s per lactation on a mainframe computer and 0.001 s on a workstation. Missing traits and fewer tests can greatly reduce processing times because matrices are smaller.

For records in progress, users may desire to know both the predicted 305-d yield and the incomplete total yield at the most recent test day, for example, 111-d yield of the cow. Additional time and memory were required to sum the covariances for these incomplete totals because of their nonstandard length. Standard 305-d yields were simpler to compute because covariances of 305-d yield with any daily yield were computed once at the beginning of the program and stored in a small vector.

### Formulas

The test day correlation matrix accounted for differing number of tests and intervals between test days for standard testing, but the variances and correlations within the matrix should be adjusted to account for other types of tests. For LER tests, the variance $\left(\sigma^2_{\mathrm{LER}}\right)$ of the reported mean was

$$\sigma^2_{\mathrm{LER}} = \Sigma_i \Sigma_j V_{ij}/N^2$$

where test days i and j are both summed over the days included in the mean, and N = number of days in the mean. To account for greater variance of a.m.-p.m. tests, diagonal elements $V_{ii}$ were increased by 0.3[(number of milkings/milkings tested) – 1]$V_{ii}$. The value of 0.3 for the constant, a, resulted in lactation weights and squared correlations equal on average to those calculated earlier by T. R. Meinert (USDA, 1995, unpublished data).

Off-diagonals $V_{ij}$ for traits measured on the same day were increased by 0.3[(number of milkings/milkings tested for either trait) – 1]$V_{ij}$. For owner-sampler testing, 0.18$V_{ii}$ was added to the diagonals, $0.18(V_{ii}V_{jj})^{0.5}$ was added to the off-diagonals for the same trait but different test days, and 0.18$V_{ij}$ was added to the off-diagonals for different traits. The constant of 0.18 resulted in an assumed squared correlation between predicted and true lactation yield of 0.75 for owner-sampler testing versus 0.97 for supervised testing (see Table 1). Corresponding DCR were 77 for monthly owner-sampler records versus 100 for monthly supervised testing.

## Lactation Curves

Lactation curves for an example cow were plotted using the unadjusted test interval method (15) (Figure 1) and best prediction (Figure 2). Best prediction resulted in a smoother curve and predicted daily yields that, on average, were slightly closer to the herd mean. The 305-d yields computed by best prediction also may be less variable than those computed by the test interval method, which does not regress toward the mean. Predicted daily yields are less variable than true daily yields because correlations between any two test days are <1.0. Predicted yields equal true yields only on days when full day, supervised tests are conducted and no measurement error is assumed. With other types of tests, such as a.m.-p.m., predicted yields on test day are regressed toward the mean and smoothed across the lactation because observed yields contain true yields plus measurement error.

For an incomplete lactation (Figure 3), best prediction extended the curve to 305 d by extrapolation; the test interval method, however, was designed only for simple interpolation between test days. Curves from prediction methods such as random regression should be similar to best prediction when extrapolating from a few points in early lactation. In contrast, the straight lines of the test interval method

TABLE 1. Lactation weights, squared correlations of true with estimated milk yields, and data collection ratings for various test plans.

| Test plan | Test days | Lactation weight | Squared correlation McDaniel (3) | Squared correlation Proposed | Data collection rating |
|---|---|---|---|---|---|
| | (no.) | (%) | | | |
| Daily | 305 | 100 | 100 | 100 | 103 |
| Labor efficient record | | | | | |
| 10-d Mean | 100[1] | 99 | . . . | 100 | 103 |
| 5-d Mean | 50[2] | 98 | . . . | 99 | 102 |
| Monthly supervised milkings | | | | | |
| All[3] | 10 | 94 | 97 | 97 | 100 |
| 2 of 3 | 10 | 89 | . . . | 95 | 97 |
| 1 of 2 | 10 | 84 | 95 | 92 | 95 |
| 1 of 3 | 10 | 77 | . . . | 88 | 90 |
| Monthly owner-sampler milkings | | | | | |
| All[3] | 10 | 57 | . . . | 75 | 77 |
| 2 of 3 | 10 | 55 | . . . | 73 | 75 |
| 1 of 2 | 10 | 53 | . . . | 72 | 74 |
| 1 of 3 | 10 | 50 | . . . | 69 | 71 |
| Bimonthly supervised milkings | | | | | |
| All[3] | 5 | 88 | 93 | 94 | 97 |
| 2 of 3 | 5 | 80 | . . . | 90 | 92 |
| 1 of 2 | 5 | 73 | . . . | 86 | 88 |
| 1 of 3 | 5 | 62 | . . . | 78 | 81 |
| Bimonthly owner-sampler milkings | | | | | |
| All[3] | 5 | 55 | . . . | 73 | 75 |
| 2 of 3 | 5 | 51 | . . . | 70 | 72 |
| 1 of 2 | 5 | 48 | . . . | 67 | 69 |
| 1 of 3 | 5 | 43 | . . . | 63 | 65 |

[1]A 10-d mean reported in each of 10 mo.

[2]A 5-d mean reported in each of 10 mo.

[3]Milk weights were obtained from all milkings on each test day.

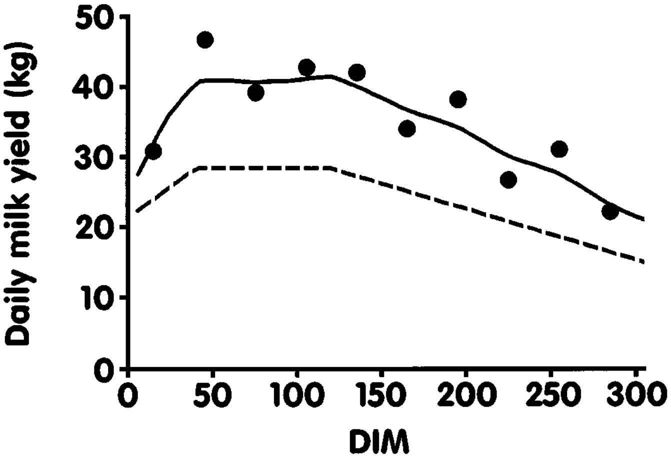


Figure 2. Example lactation plotted by best prediction (———) and compared with contemporary mean (– – –) where ● = supervised milk weight.

connect each observed yield but may produce poor curves if observations are far apart. Curves by best prediction are very flexible and can adapt to almost any data pattern; shapes of the curves by random regression are limited by the number of parameters. Also, observations from each test day can be weighted according to their precision both in best prediction and in random regression, but precision of test day data is not considered in the test interval method.

### Accuracy

Squared correlations of estimated and true 305-d yields, DCR, and lactation weights for 19 current or anticipated milk recording plans are in Table 1. For comparison, squared correlations that were derived from McDaniel (3) are included for the plans he studied. Milk records from LER plans were only slightly less accurate than daily testing. Monthly testing had a squared correlation of 0.97 with daily testing, which agrees with 0.97 derived from the study of McDaniel (3). Lactation weights follow a pattern similar to that for squared correlations but tend to be lower. Proposed weights are less than previous weights (14) because each testing plan now is compared with daily rather than monthly testing. Squared correlations were converted to DCR by dividing by the 0.97 value for traditional testing. Thus, DCR are higher than the squared correlations by a factor of 1.03.

Matrix algebra gives correlations of estimated and true lactation yield that are more precise than those obtained by examining samples of data if the (co)variance structure and distribution of daily yield are actually known. However, lactation curves and (co)variances may differ for cows of different breed, parity, and time period. Further research is needed to compare the results of random regression and other estimation procedures to best prediction.

## CONCLUSIONS

Best prediction provided a flexible method to calculate 305-d yields and accuracies of those yields as measured under many different test plans. Computations were more difficult than for the test interval method but are affordable. Multiple-trait equations allowed missing yields for fat and protein to be estimated from test day milk yield. The number of test days and the length of test intervals were automatically accounted for by the correlations between one daily yield and all other daily yields.

Increased measurement errors were assumed for a.m.-p.m. and owner-sampler testing. A mean of consecutive daily yields has less error variance than an individual test day yield, and thus data from LER plans may be more accurate than data from other plans. The highest lactation weight should represent 305 supervised test days and 305 samples instead of 10 as in the past. Heritability of lactation yield should increase slightly with the daily measurements possible with LER. Genetic evaluations may soon combine information from the many individual test day yields instead of lactation yields by BLUP. Such systems require many simultaneous equations. Until then, multiple-trait predictions, graphs, and measures of

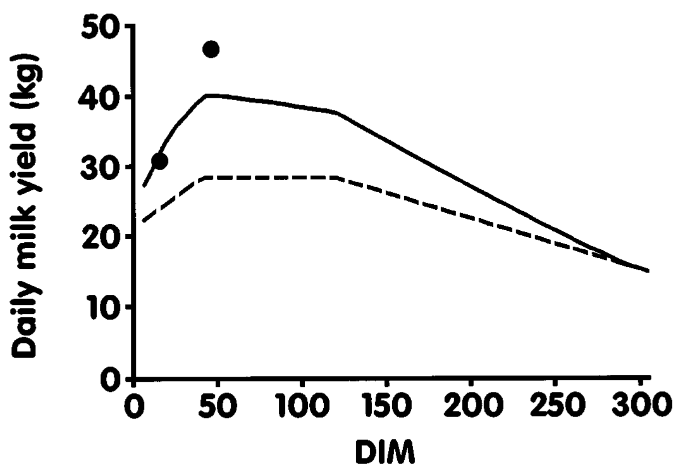


Figure 3. Example lactation in progress plotted by best prediction (———) and compared with contemporary mean (– – –) where ● = supervised milk weight.

accuracy can be calculated one lactation at a time by best prediction.

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## REFERENCES

1 Johnson, D. L. 1996. Estimation of lactation yield from repeated measures of test day yields. Proc. N.Z. Soc. Anim. Prod. 56:16.
2 Jones, L. P., and M. E. Goddard. 1990. Five years experience with the animal model for dairy evaluations in Australia. Proc. 4th World Congr. Genet. Appl. Livest. Prod., Edinburgh, Scotland XIII:382.
3 McDaniel, B. T. 1969. Accuracy of sampling procedures for estimating lactation yields: a review. J. Dairy Sci. 52:1742.
4 Meyer, K. 1985. Genetic parameters for dairy production of Australian Black and White cows. Livest. Prod. Sci. 12:205.
5 Norman, H. D., P. M. VanRaden, J. R. Wright, and T. R. Meinert. 1996. Mathematical representation of relationships between daily milk yields. J. Dairy Sci. 79(Suppl. 1): 143.(Abstr.)
6 Ptak, E., and L. R. Schaeffer. 1993. Use of test day yields for genetic evaluation of dairy sires and cows. Livest. Prod. Sci. 34: 23.
7 Reents, R., J.C.M. Dekkers, and L. R. Schaeffer. 1995. Genetic evaluation for somatic cell score with a test day model for multiple lactations. J. Dairy Sci. 78:2858.
8 Sargent, F. D., V. H. Lytton, and O. G. Wall, Jr. 1968. Test interval method for calculating Dairy Herd Improvement Association records. J. Dairy Sci. 51:170.
9 Schaeffer, L. R., and J.C.M. Dekkers. 1994. Random regressions in test-day production in dairy cattle. Proc. 5th World Congr. Genet. Appl. Livest. Prod., Guelph, ON, Canada 18:443.
10 Schaeffer, L. R., and J. Jamrozik. 1996. Multiple-trait prediction of lactation yields for dairy cows. J. Dairy Sci. 79:2044.
11 Shook, G. E., L. P. Johnson, and F. N. Dickinson. 1980. Factors for improving accuracy of estimates of test-interval yield. DHI Lett. 56:4.
12 Trus, D., and L. G. Buttazzoni. 1990. A multiple trait approach to modelling the lactation curve. Proc. 4th World Congr. Genet. Appl. Livest. Prod., Edinburgh, Scotland XIII:492.
13 VanRaden, P. M., H. D. Norman, R. L. Powell, and G. R. Wiggans. 1996. Changes in USDA-DHIA genetic evaluations (July 1996). AIPL Res. Rep. CH6(7-96). Anim. Improvement Progr. Lab., USDA-ARS, Beltsville, MD.
14 VanRaden, P. M., G. R. Wiggans, and C. A. Ernst. 1991. Expansion of projected lactation yield to stabilize genetic variance. J. Dairy Sci. 74:4344.
15 Voelker, D. E. 1985. History of dairy recordkeeping. Natl. Coop. DHI Progr. Handbook, Fact Sheet A-2. Ext. Serv., USDA, Washington, DC.
16 Wiggans, G. R., and M. E. Goddard. 1996. A computationally feasible test day model with separate first and later lactation genetic effects. Proc. N.Z. Soc. Anim. Prod. 56:19.
17 Wiggans, G. R., and P. M. VanRaden. 1991. Method and effect of adjustment for heterogeneous variance. J. Dairy Sci. 74:4350.