

Analysis of Productive Life in Swiss Brown Cattle

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ABSTRACT

An analysis of productive life of Swiss Brown cattle was performed using a mixed survival model based on Cox regression. Data included 52,862 daughters of 297 sires. The length of productive life was observed from May 1, 1985 through August 15, 1995. Records on cows that were still alive in the end of study (32.4%) were treated as censored. The probability of being culled (hazard) was defined as a product of a baseline hazard function and a function of explanatory variables. In addition to sire effects, the model included effects of age at first calving and the time-dependent variables herd by year, lactation number, stage of lactation, and milk production within the herd to account for culling because of low production. Solutions for fixed effects indicated a higher probability of being culled for primiparous cows, for cows in the end stage of lactation, and for cows with low production. The impact of censoring on the accuracy of estimation was investigated by computing the rank correlations between the estimated transmitting abilities (ETA) of sires using a simplified model from uncensored data (reference) and the ETA from several different data files with an increased proportion of censored records. The rank correlations among sire ETA decreased as number of daughters per sire decreased and as the proportion of censored records increased. The maximum number of censored records that is acceptable to obtain accurate results is 30 to 40%. The acceptable proportion of censored records would be higher if the reference ETA were obtained on a larger data file using daughters of old sires. (**Key words:** productive life, survival analysis, estimated transmitting ability, Swiss Brown cattle)

INTRODUCTION

A long productive life is one of the most important components of dairy cow profitability. The largest

effect of longer productive life is decreased replacement costs. Longer productive life also leads to a higher proportion of cows that are in later, high producing lactations. An increase in length of productive life from three to four lactations increases milk yield per lactation and profit per year by 11 to 13% (16).

Ducrocq (2) defined two types of productive life that are of interest to breeders. True productive life is the actual productive life and mainly depends on productivity. Functional productive life depends on the ability of the cow to avoid culling for involuntary reasons such as sterility or disease. For breeding programs, genetic evaluation for functional productive life is more relevant because that evaluation brings new information that complements the evaluation for production traits.

However, the statistical analysis of productive life and the prediction of breeding values for this trait present a problem because some of the cows are still alive at the end of the study period. The records of those cows are referred to as censored because only the lower bound of herd life is known for those animals. To exclude those records from the analysis, or to consider them as exact, would bias results (4). Moreover, traditional linear models of analysis are inappropriate because they do not accommodate data on productive life (15).

Survival analysis, which includes statistical methods developed primarily for medical research, seems to be more appropriate for genetic evaluation of productive life (14). Survival analysis enables the use of complete available information on uncensored as well as censored records, provides a proper statistical treatment of censored records, and accounts for the nonlinear characteristics of productive life data (9).

One of the most frequently used models in the analysis of productive life is a proportional hazard model, which also is known as a Cox regression (13). This model is based on hazard rate, $\lambda(t)$, which defines the instantaneous rate of failure, or, if applied to dairy cows, the probability of being culled at time t , given that a cow is alive immediately prior to t . The probability that a cow survives longer than a specified

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time t is given by a survivor function, $S(t)$. $S(t)$ can also be interpreted as the proportion of cows still alive at time t . The hazard rate, $\lambda(t)$, is described as a product of a baseline hazard function, $\lambda_0(t)$, representing the aging process and a function of explanatory variables that supposedly influence culling process. The effect of these variables can be estimated independently from the baseline hazard function using a semiparametric estimation procedure (1). Thus, the baseline hazard function can be completely arbitrary. Therefore, the Cox model is appealing for the analysis of data on productive life, for which the shape of the survivor function is not always known with certainty (9).

The Cox model assumes proportional hazard (i.e., the ratio of hazard functions for two animals is constant over the entire time period) (13). This assumption does not hold for productive life data, because the hazard for individual cows varies differently over time, depending on current culling criteria. However, the model can be generalized by introducing time-dependent explanatory variables that account for variations in culling policy over time, such as herd by year and lactation number. In this way, the time axis is partitioned into several shorter intervals. The assumption of proportional hazard holds within each interval, but the hazard ratio changes from one interval to another. With time-dependent variables in the model, several levels of a fixed effect can be associated with a single record for productive life.

In addition, Cox regression can be extended to include random effects (e.g., genetic effects), which lead to mixed survival models, known as "frailty" models (5). Mixed survival models can be used for genetic evaluation of sires and cows for the length of productive life.

When used for genetic evaluation, the predictive power of survival analysis strongly depends on the proportion of censored records. The accuracy of evaluation would be expected to decrease as the proportion of censored records increases. Therefore, it is necessary to determine whether the results obtained from censored records of daughters of young sires are sufficient to predict the productive life of these cows or new daughters of the sire.

For routine sire evaluation, Egger-Danner (9) considered survival analysis as better than traditional BLUP evaluation for stayability.

The aims of the present study are to analyze genetic and environmental effects on herd life in Swiss Brown cattle by means of survival analysis and to investigate the impact of censored records for genetic evaluation.

MATERIALS AND METHODS

The data included 52,862 daughters of 297 Swiss Brown sires. Cows that first calved between 1985 and 1994 were considered. Their productive life was followed from May 1, 1985 to August 15, 1995. Sires were required to have at least 20 daughters with uncensored records (i.e., with complete productive life). The length of productive life was defined as the number of months between the first calving and the culling date. Cows with missing culling dates were assumed to have been culled if the milk recording was abandoned at least 8 mo before the end of the study (August 15, 1995). All other cows were assumed to be alive at the end of study and were treated as censored. The proportion of censored records was 32.4%.

The analysis was carried out using the following mixed model based on Cox regression:

$$\lambda(t) = \lambda_0(t) \exp(s_i + HY_j(t') + AFC_k + LN_l(\tau) + LS_m(\tau) + MP_n(\tau))$$

where

- $\lambda(t)$ = hazard function (probability of being culled) at time t ,
- $\lambda_0(t)$ = baseline hazard function, related to the aging process of the cows,
- s_i = random effect of sire i ,
- $HY_j(t')$ = fixed time-dependent effect of herd by year combination j ,
- AFC_k = fixed effect of class k of age at first calving,
- $LN_l(\tau)$ = fixed time-dependent effect of lactation l ,
- $LS_m(\tau)$ = fixed time-dependent effect of stage of lactation m , and
- $MP_n(\tau)$ = fixed time-dependent effect of within-herd production level n .

Three time scales were defined: t , with the origin on the date of first calving; t' , with the origin on May 1, 1985; and τ , with the origin on the calving date corresponding to the current lactation.

Sire effects were assumed to be normally distributed. Heritability of productive life was assumed to be 0.08, as estimated in a previous study using the REML method with a sire model (18). The relationship matrix included sires and maternal grandsires of 297 sires with daughters in the data. The relationship matrix comprised 447 individuals.

Herd by year effects were assumed to be piecewise constant with changes on May 1 of each year. Only herds with a minimum of 20 completed lactations per year were considered. Year effects were defined from

May 1 to April 30 to correspond with the milk quota period in Switzerland. Many producers extensively cull their cows at the end of each quota period in order to avoid overproduction. In total, 9882 herd by year effects were estimated.

Four classes of age at first calving were considered: ≤ 30 mo, 31 to 33, 34 to 36, and ≥ 37 mo. The effect of lactation number was assumed to be piecewise constant; changes occurred at the beginning of each lactation. Lactations 1, 2, 3, and ≥ 4 were considered. Effect of stage of lactation was also piecewise constant, with changes occurring at 0, 60, and 180 d after calving and at the beginning of dry period. The effect of stage of lactation was included to account for different physiological phases during the reproductive cycle of a cow, which were supposed to coincide with changes in culling intensity. Effect of production level within a herd was piecewise constant; large increases occurred at the beginning of each lactation. The production levels were defined according to the 305-d first lactation equivalent milk yield of each cow, expressed as the percentage of the herd mean. Within each lactation, five production levels were distinguished; $< 80\%$, 80 to 94%, 95 to 104%, 105 to 120%, and $> 120\%$ of the mean herd production. The effect of within-herd production level was included to account for culling for low production and to enable the estimation of the functional productive life.

The estimation of the baseline hazard function ($\hat{\lambda}_0$), baseline survivor function (\hat{S}_0), and solutions for fixed (\hat{b}) and sire (\hat{s}) effects were based on maximum likelihood techniques. The analysis was performed using the software package The Survival Kit (6).

The impact of censoring on sire ETA for productive life was studied using Spearman's rank correlations between sire breeding values estimated from data files with different proportions of censored records. For this purpose, the model used for analysis was simplified by excluding the effect of age at first calving from the full model and assuming no relationships among sires. The reference data were constructed by deleting all censored records from the original data. Sire effects that were estimated from these data were used as reference ETA. The analysis was then repeated 10 times with truncated data files, each time assuming August 15th of the previous year as the end of data collection (i.e., August 15, 1995 through August 15, 1986). Records on cows culled after this date were treated as censored. After each step, rank correlations among sire effects were calculated. These correlations were used to compare early sire evaluation with the reference. To obtain more detailed information about the impact of censoring on the ETA,

TABLE 1. Length of productive life for uncensored and censored records.¹

Productive life	Uncensored	Censored
	(mo)	
Mean	29.5	38.5
Minimum	0.7	2.7
Maximum	118.6	124.3

¹There were 35,739 uncensored and 17,123 censored records.

sires were divided into four groups by the number of uncensored daughters in the data file with the highest proportion of censored records. Rank correlations were then calculated separately for each group. To account for different group size, the bootstrap sampling method was applied (19). From each group, a sample of 10 randomly chosen sires was drawn, and rank correlations were calculated for these sires. The procedure was repeated 1000 times. Rank correlation coefficients, averaged over 1000 replicates, were used to judge the ETA.

RESULTS AND DISCUSSION

The number of cows and mean, minima, and maxima for the length of productive life using uncensored and censored records are given in Table 1. The mean length of productive life was 29.5 mo for uncensored records and 38.5 mo for censored records. The mean length of productive life for all cows in the analysis (uncensored and censored) was 32.4 mo (SD = 25.5 mo).

Baseline Survivor and Hazard Functions

Figure 1 shows the estimated baseline survivor function ($S_0(t)$) for all cows in the analysis. Baseline survivor function is the fraction of cows that would be still alive t months after first calving if culling was not influenced by the covariates considered in the analysis model.

The estimated baseline hazard function is shown in Figure 2. Baseline hazard function shows the risk of being culled at a time t , which depends on the age of a cow. The risk of being culled increases as cows get older (17). Larger deviations in estimated culling risk at advanced age occur primarily because fewer old cows were included in the data.

Likelihood Ratio Test

The significance of the explanatory variables was tested using a likelihood ratio test for large samples.

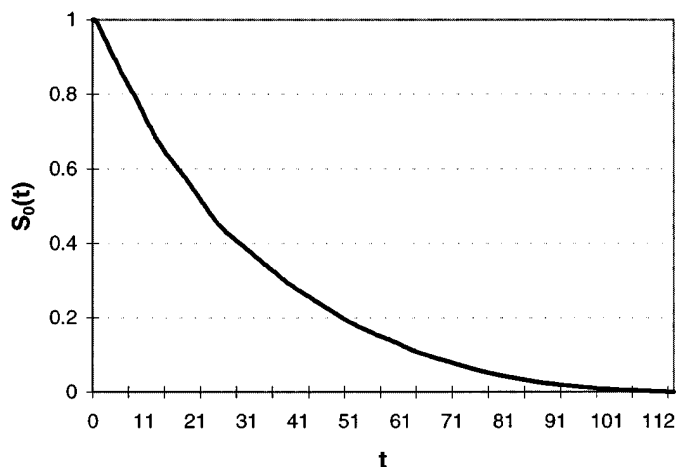


Figure 1. Estimated baseline survivor function [$S_0(t)$] for all cows in the analysis. t = Months after first calving.

Twice the change in log likelihood that was induced by the inclusion of a new effect is compared with a chi-square distribution with ν degrees of freedom, where ν is number of added estimated effects (4). Table 2 shows the results of the likelihood ratio test.

All effects included in the model were significant ($P < 0.001$). The most dramatic changes in log likelihood were observed after the effects of production level within the herd and of herd by year were included. The change in log likelihood that was associated with the age at first calving was very small compared with that of the other effects. Therefore, this effect was considered to be unimportant. Ducrocq (4) also reported negligible effects of the age at first calving on the length of productive life in dairy cows.

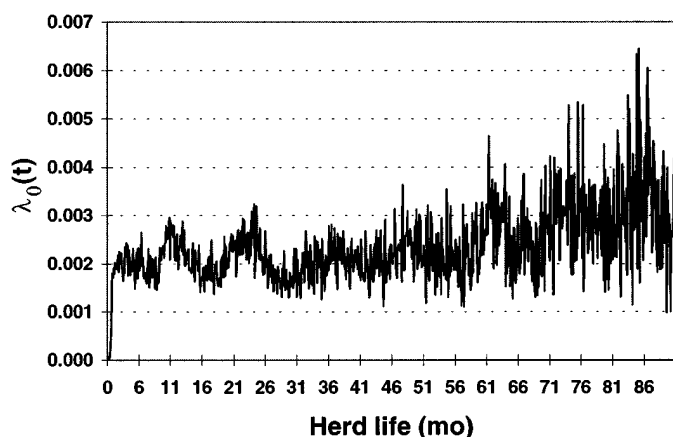


Figure 2. Estimated baseline hazard function [$\lambda_0(t)$] for all cows in the analysis.

TABLE 2. Results of likelihood ratio test.¹

Effect ²	-2 Change in log likelihood	df
s	1257	447
s, HY	11,367	9922
s, HY, AFC	19	3
s, HY, AFC, LN	3251	3
s, HY, AFC, LN, LS	8961	3
s, HY, AFC, LN, LS, MP	22,850	19

¹All effects in the model were significant ($P < 0.001$).

²s = Effect of sire, HY = Effect of herd by year, AFC = effect of age at first calving, LN = effect of lactation number, LS = effect of stage of lactation, and MP = effect of relative milk production within the herd.

Solutions for Fixed and Sire Effects

To facilitate interpretation, the results are expressed as relative culling rate, defined as the ratio between the estimated risk of being culled under the influence of a certain environmental or genetic effect [$\exp(\hat{b})$], and mean risk, which is normally set to 1 [$\exp(0)$]. For example, a daughter of a sire with a relative culling rate of 2 has a probability of being culled that is twice that of a daughter of an average sire under average environmental conditions (3). For selection aimed at improving productive life in the population, lower culling rates are desirable. For fixed effects that are time dependent, the level with the highest number of uncensored records was set to 0 and assumed to be the average risk. This level was used for comparison with all other levels of the effect.

Figure 3 shows the herd by year effects averaged over all herds. In general, the mean risk tends to increase slightly from one year to the next. This increase could be explained by more intensive selection in the recent years (11). However, these results should be interpreted with caution. The lower estimated culling risk in the first 2 yr is partly a consequence of fewer observations and, especially, fewer failures, because the estimation of culling risk in this period was based almost solely on first lactating cows, which certainly influenced the results and might have led to estimates that were biased downward (4).

In contrast to results of some previous studies (4, 6, 7) in which lactation number and stage of lactation were estimated jointly (lactation number by stage of lactation), in this present study, these effects were treated independently. The effects of lactation number and stage of lactation on relative risk are presented additively (lactation number and stage of lactation) in Figure 4. Ducrocq (2) proposed to separate these two effects in order to avoid the

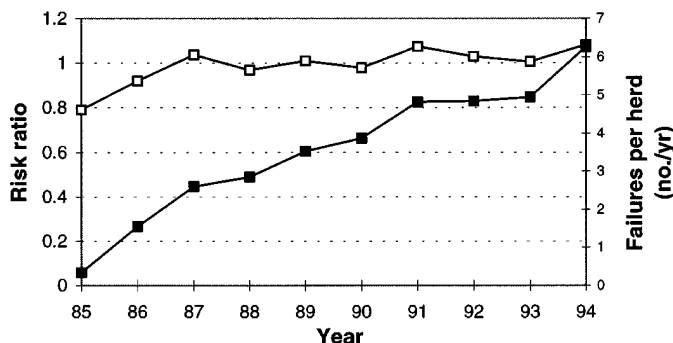


Figure 3. Estimates of the effect of herd by year (□) and the number of failures per herd and year (■), averaged over herds.

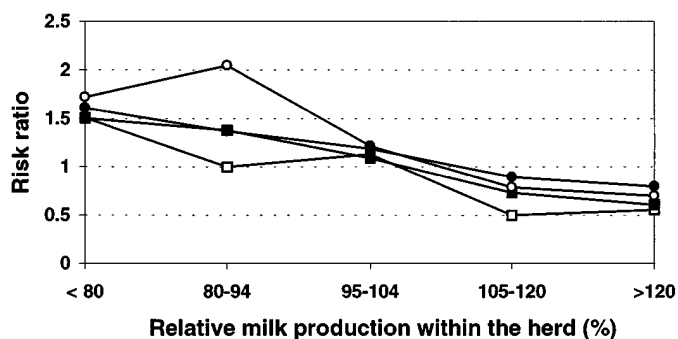


Figure 5. Estimates of the effects of relative milk production within the herd in lactations 1 (□), 2 (■), 3 (○), and ≥ 4 (●).

problem of not being able to estimate the effects of stage of lactation for first calving cows when all cows are in the first, second, or third stage of lactation at the same time, and, therefore, the comparison with other lactation number by stage of lactation effects is not possible. Moreover, no interactions have been found between lactation number and stage of lactation (2). Under this assumption, separation of the effect of lactation number from that of stage of lactation enables a direct comparison of animals in different stages of lactation.

The relative culling rate is larger during the first lactation than during later lactations. Within each lactation, the relative culling rate also varies significantly. As expected, a cow finishing a lactation is at much higher risk of being culled than an identical cow in early or midlactation, because the dry period is the period of the most intensive selection in dairy herds. The risk of being culled also increases up to 60 d after calving, which might be due to a higher incidence of health disorders during early lactation.

Figure 5 illustrates the influence of milk production within a herd on culling; within each lactation,

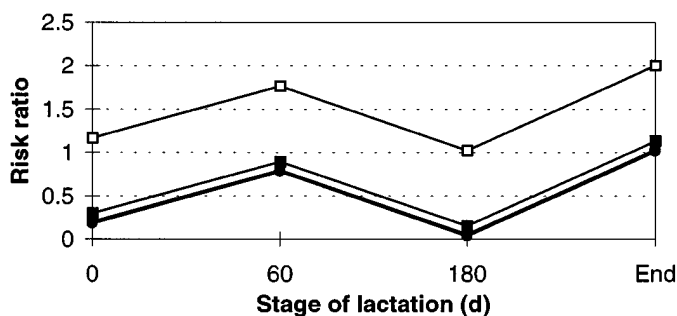


Figure 4. Estimates of the effects of lactation number and stage of lactation on the risk ratio. Lactations 1 (□), 2 (■), 3 (○), and ≥ 4 (●).

low producing cows are at higher risk than their herdmates with average production, and high producing cows are less likely to be culled. This lower risk of culling might also be caused by the preferential treatment of these top cows (12).

The ETA for 297 sires, estimated from the original data with the full model, averaged 0.9 and ranged from 0.3 to 3.9. These ETA can be interpreted as a relative culling rate of daughters of a particular sire compared with daughters of all other sires with the same productive life and under average environmental conditions. Sire effects can also be determined indirectly by computation of an "expected survivor curve". Figure 6 shows such curves for three sires with different relative culling rates, assuming average environmental conditions. These curves can be used to interpret a sire transmitting ability for productive life as the fraction of daughters expected to achieve a certain length of productive life (4).

Impact of Censoring on ETA

The number of sires and daughters with uncensored and censored records in each of 10 analyses are given in Table 3.

The first data file without censored records was used to obtain reference solutions for sire effects, using the simplified model, as described. This data file and the corresponding estimates from the analysis are referred to as reference in further discussion.

One hundred and thirty-eight sires of the 297 had daughters in all data files. For these sires, Spearman's rank correlations among estimates in 10 successive yr are given in Table 4.

The rank correlations among sire ETA decreased when the period of study was shorter and when the proportion of censored records was higher. When the proportion of censored records increased by 1%, the

TABLE 3. Number of sires and cows, number of uncensored and censored records, and percentage of censored records in 10 successive analyses.

Year	Sires	Cows	Uncensored	Censored	Censored
			records	records	records
			(no.)	(%)	
1995	297	35,739	35,739	0	0
1994	297	35,346	30,858	4488	14
1993	297	33,706	25,893	7813	23
1992	295	31,092	20,903	10,189	33
1991	284	27,786	15,950	11,836	43
1990	273	24,008	11,812	12,196	51
1989	251	19,980	8114	11,866	59
1988	217	15,454	5111	10,343	67
1987	177	10,586	2478	8108	77
1986	138	5406	572	4834	89

rank correlations decreased by 0.01. The correlation between sire ranks from the reference and from the earliest analysis with 90% censored records was only 0.15. Reasonably high rank correlation of 0.8 to 0.9 were obtained up to 33% censored records.

Rank correlation coefficients, calculated separately within each of four groups of sires on repeated samples of 10 randomly drawn sires, are shown in Table 5.

Compared with the reference, large differences in ranking occurred in sires with no or few uncensored daughters in the earliest analysis. For sires with no uncensored daughters, culling rates were extremely low. The average deviation from the reference, expressed as a percentage of the reference breeding value, was -59%. Therefore, most of these sires were ranked much higher by the earliest analysis than by the reference analysis. However, the lower rank in the reference was probably because young sires in the reference data were represented only with daughters with extremely short productive life (censored records were excluded). Therefore, reference ETA for those sires might be biased. However, sires with ≤ 9 uncensored daughters in the earliest analysis were ranked

considerably lower than the same sires in the reference. Sires with ≥ 10 uncensored daughters in the earliest analysis, although biased upward were ranked fairly correctly (rank correlation 0.72 compared with the reference) and with smaller variation among individual samples (average standard deviation of the correlation coefficient was 0.11). These sires had also a larger total number of daughters.

A similar study has been conducted by Egger-Danner et al. (10), who compared the ranking of sires according to ETA from the full data file without censored records and from truncated data with a different proportion of censored records. Rank correlations between ETA on the full data file and on the censored data were lower as the proportion of censoring increased. However, they (10) found that rank correlations were larger than those presented in this paper, probably because of the different method of sampling. In the present study, instead of allowing all cows to reach a certain productive life and having the same cows in all analyses, a method of sampling that better suits the real life situation was applied. Sires were evaluated using information on daughters that became available after a certain time. In the earliest

TABLE 4. Spearman's rank correlations among ETA for 138 sires in 10 successive yr.

Year	1995	1994	1993	1992	1991	1990	1989	1988	1987	1986
1995	1.00									
1994	0.93	1.00								
1993	0.87	0.95	1.00							
1992	0.78	0.85	0.90	1.00						
1991	0.63	0.68	0.76	0.80	1.00					
1990	0.51	0.54	0.61	0.66	0.85	1.00				
1989	0.46	0.50	0.58	0.64	0.77	0.91	1.00			
1988	0.35	0.40	0.45	0.54	0.59	0.70	0.78	1.00		
1987	0.26	0.34	0.43	0.48	0.53	0.68	0.76	0.68	1.00	
1986	0.15	0.23	0.22	0.28	0.23	0.28	0.36	0.23	0.36	1.00

TABLE 5. Spearman's rank correlations among sire ETA from the earliest analysis and from the reference data.¹

Uncensored daughters	Sires	r	S _r	Diff	Dau1	Dau2
(no.)				(%)		(no.)
0	71	0.40	0.28	-59	120.0	0.0
1-3	36	0.25	0.30	+55	158.4	1.8
4-9	15	0.18	0.17	+84	120.6	5.9
≥10	16	0.72	0.11	+68	353.6	26.2

¹r = Spearman's rank correlation coefficient, S_r = standard deviations of r, Diff = relative differences in risk ratio, Dau1 = mean number of uncensored daughters in the reference, and Dau2 = mean number of uncensored daughters in the earliest analysis. The values were obtained from the sample of 10 randomly chosen sires and averaged over 1000 replicates.

analysis, which was conducted only about 15 mo after the beginning of data collection, the data file was small (~5000 cows), and consequently, the amount of information was proportionally low. In the later analyses, the data file was expanded to include later born daughters of the same sires as well as daughters of younger sires to increase the basis for comparison. Variations in the sample size, especially the low number of records in the earliest analysis, and the fact that reference breeding values for young sires were based only on daughters with extremely short productive life, had a significant impact on the results.

CONCLUSIONS

Survival analysis is an appropriate method to use for analysis of productive life data, because survival analysis fully utilizes the available information from uncensored as well as from censored records. The most important effects that influence productive life

are lactation number, stage of lactation, and relative milk production within the herd. Cows in first lactation have a higher risk of culling than do older cows. Also, dry cows and cows with milk production that is lower than the herd average are more likely to be culled. When the model of analysis includes time-dependent covariates that account for changing environment and culling for low production, survival analysis can be used for genetic evaluation for functional herd life.

However, according to the results from this study, at least 60 to 70% of the data should be uncensored. A sufficient number of daughters per sire is also required. With no or only a few uncensored records and few daughters per sire, ETA and ranking of sires are biased. It should also be taken into account that early analyses in this study were conducted with considerably fewer records than were in the reference data and that the reference data for young sires consisted only of daughters with short productive life, which is likely to cause lower rank correlations.

Further research, based on larger data or simulated records, is needed to determine exactly the effects of censoring on ETA for productive life. Future investigation should also concentrate on the impact of including relationships among sires, which ought to improve the accuracy of genetic evaluation, especially for young sires. Moreover, inclusion of certain auxiliary traits such as type score, physiological parameters, or genetic markers in the survival analysis should also be considered.

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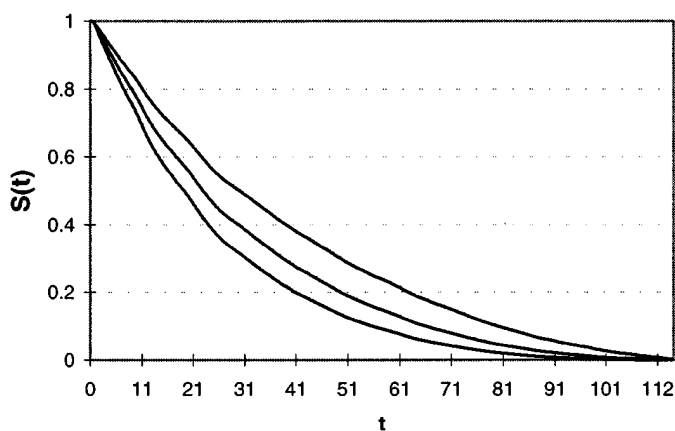


Figure 6. Expected survivor curves [S(t)] for three sires with different values for relative culling rate s_i . From top to bottom: $s_i = 0.75$, $s_i = 1.00$, and $s_i = 1.25$. t = Months after first calving.

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