Use of Profit Equations to Determine Relative Economic Value of Dairy Cattle Herd Life and Production from Field Data

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ABSTRACT

Profit equations or functions that reflect the realized profitability of cows have been used in the literature to determine the relative importance of different variables such as milk yield and herd life. In all profit equations, the opportunity cost of postponed replacement, which reflects the profit sacrificed on an average replacement cow by keeping the cow, has been omitted.

In this study, three profit equations were compared: total lifetime profit; total lifetime profit accounting for opportunity costs, and profit per day of herd life. Linear regression was used to determine the relative value of first lactation production and herd life on profitability in simulated data.

When accounting for opportunity costs, the value of an additional day of herd life was equal to that of a 4.0 kg higher first lactation milk production. The relative value of herd life was overestimated by 260% when opportunity costs of postponed replacement were not accounted for. The need to account for the opportunity cost of postponed replacement in calculating economic weights is clearly demonstrated.
(**Key words**: profit equation, herd life, profitability)

Abbreviation key: **Dfl** = Dutch florins, **OPCOST** = opportunity costs of postponed replacement, **PPD** = profit per day of herd life, **PROFOP** = total lifetime profit accounting for opportunity costs, **PRTOT** = total lifetime profit.

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INTRODUCTION

The breeding goal for dairy cattle is to increase economic merit of cows. When several traits contribute to economic merit, economic weights are used to combine the estimated breeding value for the component traits into an estimated breeding value for economic merit of animals for selection purposes (8). Economic weights are also essential in predicting revenues of a breeding program. Methods for calculating economic values can be grouped into two categories: normative approaches, also referred to as bioeconomic modeling, and positive approaches, which involve analysis of field data (4, 7).

Dynamic programming (12, 13, 17, 19) is one of the modeling methods used to determine the relative economic value of production and dairy cattle herd life. This technique is based on replacement principles (3). The optimum time for replacement of a cow is determined by comparing its expected marginal net revenues with the opportunity cost of postponed replacement. The average net revenue from an average cow during her lifetime can be interpreted as the opportunity cost of postponed replacement, in which case the value of herd life is equal to the net revenues during the additional day of herd life corrected for the average net revenues per day of an average animal. When opportunity costs are not accounted for, the value of herd life is determined by the net revenues during the additional day; this implies that the number of cows in the herd is considered to vary with the length of herd life.

Profit equations or functions that reflect the realized profitability of cows have been used to determine the relative importance of different variables such as herd life and production based on field data (1, 2, 6, 9, 15). Profit equations have included revenues and costs over a fixed time period or the total herd life of the cow. Equations differ mainly in completeness and estimation methods [see review (11)]. The opportunity costs of postponed replacement are not considered. Both total profit and average profit per day have been used to reflect the profitability of the cow. Meaningful arguments to select one of these criteria are lacking.

In this paper, the impact of replacement principles on the use of profit equations to determine the relative economic weights of herd life and production is discussed. Differences between profit equations in the relative value of production and herd life are derived theoretically and illustrated using simulated data.

MATERIALS AND METHODS

Data Simulation

A stochastic simulation model was used to calculate production, herd life, revenues, and costs of cows. The model was based on the dynamic programming model described by Van Arendonk and Dijkhuizen (19). In this model, cows were described by state variables and maximum number of distinct values: lactation number, 12; stage of lactation, 16; time of conception, 6; and milk production during the present, 15 and previous lactations, 15. Lactation number of the cow ranged from 1 to 12. Fifteen milk yields for each lactation number were considered based on a within-herd and within-age coefficient of variation of 12%. Conditional probabilities related milk yield in the current lactation to milk yield in the next lactation. The repeatability of milk yield was assumed to be .55 for adjacent records and .50 for records 2 yr apart. The probabilities of transition to each yield and the limits of each class based on a normal distribution are given by Van Arendonk (17). Culling of cows could take place at monthly intervals, and insemination could take place from 2 to 7 mo after calving. The probabilities of involuntary culling and conception were taken from Van Arendonk and Dijkhuizen (19). In the simulation model, involuntary culling, conception, and lactation production were treated as stochastic variables.

Optimum insemination and replacement policy was determined by dynamic programming (19). The optimum insemination and voluntary culling decisions for every possible state of the cow were used to account for the farmer's influence on herd life of cows.

The costs, revenues, and production from first calving until the time of culling were calculated as described by Van Arendonk (16). This represented Black and White cows in The Netherlands at the normalized price level for 1981 to 1982. See Table 1 for a summary of prices and production parameters.

TABLE 1. Prices and production parameters used in dynamic programming and simulation.

Prices, Dfl[1]	
Milk fat, kg	10.20
Milk protein, kg	9.30
Base price of milk, 100 kg	–5.60
Calves, kg	10.85
Roughages, MJ NE[2]	.045
Concentrates, MJ NE[2]	.078
Carcass weight, kg[2]	7.20
Price of replacement heifer	2500
Mature equivalent production 8 yr	
Milk, kg	6500
Fat content, %	4.10
Protein content, %	3.35

[1]1 Dutch florin (Dfl) = $.45 US.

[2]Price per kilogram of carcass weight for heifer 7 mo in lactation.

Profit Equations

For each cow in the simulated data, profitabilities were calculated according to three profit equations. In Equation [1], the profitability of an individual cow from first calving until culling from the herd (PRTOT) was calculated as

$$\text{PRTOT} = \text{REVENUES} - \text{COSTS} \quad [1]$$

where REVENUES were from milk, calves, and carcass value, and COSTS were from feeding, housing, veterinary costs, interest, and replacement heifer costs. Labor costs are excluded, so the net revenues form the compensation for the supplied labor.

In Equation [2], the cow's profit was expressed per day of herd life (PPD):

$$\text{PPD} = \text{PRTOT/HERDLI} \quad [2]$$

where HERDLI is herd life (age at culling – age at first calving) in days.

In Equation [3], the opportunity costs (OPCOST) of postponed replacement are considered in the total profitability (PROFOP):

$$\text{PROFOP} = \text{PRTOT} - \text{HERDLI} \times \text{OPCOST} \quad [3]$$

The OPCOST was calculated as the average PRTOT divided by the average HERDLI measured in days, i.e., the average net revenue per day per cow in the herd. In this study, the same OPCOST can be used for all cows because no genetic improvement was simulated and all cows were in the same herd.

Regression Model

Linear regression coefficients were calculated to determine the relative contribution of standardized 305-d milk production in first lactation and herd life to PRTOT, PPD, and PROFOP.

RESULTS

Illustration

Differences between the three profit equations are illustrated for a simplified situation in Table 2. The OPCOST were arbitrarily set equal to 650/yr. The three measures of profit, PRTOT, PPD, and PROFOP, clearly give different rankings for the four cows. Also, an identical ranking for PROFOP can be obtained by allocating the average return of an average cow (OPCOST = 650) to the years after culling for culled cows (i.e., yr 3 and 4 for cows B and C). In calculating PROFOP, we took into account that another cow would enter the herd after culling a cow. In calculating PRTOT, however, we assumed that no replacement would enter the herd.

From replacement theory (20), we know that only in situations of identical replacement can average income be used for comparing profitability of items that differ in length of productive life. The assumption of identical replacement means that every cow needs to be replaced by a heifer having exactly the same characteristics as the cow being replaced. The assumption of identical replacement does hold when comparing the profitability of average cows from different breeds. Within a breed, however, there are no systematic differences between heifers available for replacement of different cows. This means that the assumption of identical replacement does not hold for cows within a breed. Therefore, PPD does not reflect profitability of cows within a breed correctly unless no variation occurs in herd life. Cows A and B in Table 2 had the same PPD, whereas the herd life differed by 2 yr. Using PPD to compare the profitability of cows A and B over a period of 4 yr implies that cow B would be replaced by a similar cow during yr 3 and 4. The average PPD from an average replacement, however, is only equal to 650/365 = 1.78.

Simulation

Means and SD for lifetime performance and profitability are given in Table 3. Mean average herd life was 44 mo, which agrees with Van Arendonk and Dijkhuizen (19). Means for PRTOT and herd life result in an average profitability, i.e., OPCOST of 2.80 Dutch florins (Dfl) per cow per day. The average PROFOP was equal to zero as expected. Standard deviation of PROFOP was 65% smaller than that of PRTOT. Average PPD was Dfl .44.

TABLE 2. Marginal profit (in Dutch florins) per year and profitability calculated from three profit equations[1] in a simplified situation with four cows that differ in herd life.

	Marginal profit in year						
Cow	1	2	3	4	PRTOT	PPD	PROFOP[2]
A	300	800	900	800	2800	1.91	200
B	400	1000			1400	1.91	100
C	300	900			1200	1.64	−100
D	200	700	800	700	2400	1.64	−200

[1]PRTOT = Total lifetime profit, PPD = profit per day of herd life, and PROFOP = total lifetime profit accounting for opportunity costs.

[2]Opportunity costs (OPCOST) are 650 yr^{-1}.

TABLE 3. Means and SD of variables (8000 cows).

Variable	$\bar{X}$	SD
Revenues, costs[1]		
Milk revenues	14,922	12,032
Calf revenues	1421	957
Carcass revenues	1705	195
Feed costs	7090	5646
Sundry costs	4707	3424
First lactation yield,[2] kg	6510	789
Herd life, mo	44	33
Profitability[3]		
PRTOT, Dfl	3752	4025
PROFOP, Dfl	0	1417
PPD, Dfl/d	.44	6.3

[1]From first calving until culling in Dutch florins (Dfl).

[2]Mature equivalent 305-d production projected for culled cows.

[3]Calculated from profit equations: PRTOT = total lifetime profit, PPD = profit per day of herd life, and PROFOP = total lifetime profit accounting for opportunity costs.

This is Dfl 2.36 lower than OPCOST. In calculating average PPD, the average profit of each cow is weighted equally, whereas in calculating OPCOST, each cow is weighted by her herd life.

The linear regression model including herd life and first lactation production explained 96, 68, and 26% of the variation in PRTOT, PPD, and PROFOP, respectively. In the simulation model, only random variation due to factors directly related to the cow were simulated. This explains the relatively high proportion of variance in PRTOT and PPD explained by the model in comparison with values in field data (15). Results from the regression model to predict PROTOT, PROFOP, and PPD are shown in Table 4. The difference in regression coefficient for herd life between PRTOT and PROFOP is equal to Dfl 2.80/d, i.e., the opportunity costs of an additional day of herd life. The relative importance of herd life to milk production was calculated as the ratio of regression coefficients. For PRTOT, the value of one additional day of herd life was equal to that of a 14.5 kg higher first lactation milk production. For PROFOP, this value was considerably lower.

DISCUSSION AND CONCLUSIONS

Profit equations used in calculating economic weights have included revenues and costs during the total herd life of individual cows and were based on field data. In all profit equations used in the literature, the OPCOST which reflects the profit sacrificed by keeping the cow, has been omitted. Therefore, in these equations it has been assumed that a cow is not replaced after leaving the herd. The regression coefficient for herd life obtained for PRTOT is overestimated by the opportunity costs, whereas the regression coefficient for production is unaffected, which holds in general (see Appendix 1). In this study, the relative value of herd life was overestimated by 260% (Table 3) when OPCOST were not considered.

The value of a longer herd life originates from 1) lower costs of heifer replacement per day, and 2) a higher proportion of cows in most profitable lactations. To show the influence of the following function for lifetime profit was used: PRTOT = $(a + bX)H - C$, where X is production, H is herd life, C are net replacement costs, and a and b are constants. The regression coefficients for the three profit equations are shown in Table 5. With PRTOT, the value of production and herd life is equal to $b\bar{H}$ and $a + b\bar{X}$, respectively. The value of herd life is equal to that of the production in the additional day, and it does not depend on the replacement costs. With PROFOP, the economic value of herd life equals $C/\bar{H}$, i.e., the average replacement cost per day of herd life. The relative value of herd life is overestimated by $1/\bar{H}$ when PPD is used instead of PROFOP.

Several authors (5, 14, 18) showed that economic weights depend on the perspective taken, on whether it is based on a unit of product or

TABLE 4. Regression coefficients for first lactation milk production and herd life in predicting PRTOT, PPD, and PROFOP.[1]

	Regression coefficient		
Trait	Production	Herd life	Relative importance[2]
PRTOT	.269	3.89	14.5
PPD	.0009	.0030	3.51
PROFOP	.269	1.09	4.04

[1]PRTOT = Total lifetime profit, PPD = profit per day of herd life, and PROFOP = total lifetime profit accounting for opportunity costs.

[2]Value of 1 extra d of herd life expressed in kilograms of production.

TABLE 5. Regression coefficients for milk production (X) and herd life (H) in predicting PRTOT, PPD, and PROFOP[1] when marginal profit per day is a linear function of production.[2]

	Regression coefficient		
Trait	Production	Herd life	Relative importance[3]
PRTOT	$b\bar{H}$	$a + b\bar{X}$	$b\bar{H}/(a + b\bar{X})$
PPD	b	$C/\bar{H}$	$C/(b\bar{H})$
PROFOP	$b\bar{H}$	$C/\bar{H}$	$C/(b\bar{H}^2)$

[1]PRTOT = Total lifetime profit, PPD = profit per day of herd life, and PROFOP = total lifetime profit accounting for opportunity costs.

[2]PRTOT = $(a + bX)H - C$, where C are net replacement costs, and a and b are constants.

[3]Value of one additional day of herd life expressed in kilograms of production.

the individual animal. These differences disappear when the herd level of input or output is restricted (18). In these cases, the size of the enterprise should be rescaled. Thus increased output achieved genetically should be evaluated against the equivalent output achieved by rescaling. Van Arendonk and Brascomp (18) showed that the economic values obtained from PROFOP were those when size was rescaled to a fixed number of cows, whereas those obtained from PRTOT were those for a fixed number of heifers entering the herd.

Economic consequences of changes in involuntary disposal rates have been determined using dynamic programming (13, 17). When variation in production among cows was taken into account, the economic weight was 50% higher than in a situation without variation in production. This increase was caused by an increase in voluntary culling of low producing cows, in other words, an adaptation of the optimum management strategy of the farmer. In a positive approach, economic weights are calculated from data realized under the given production circumstances. Therefore, this approach cannot account for the influence of changes in optimum management with changes in genetic level. In addition, it does not offer the opportunity to study the influence of changes in the production level or of prices on the economic weights. In a normative approach, which can be used for the latter cases, knowledge is needed of traits and their relations to allow modeling of the economic merit. For secondary traits such as herd life, these relations are only partly known. For those traits, a positive approach can be useful, provided that information is given on the components that contribute to the economic weights.

When PROFOP is used on field data to estimate the economic weights of traits, the OPCOST must be known. The level of opportunity costs can be shown to have a significant effect on the relative value of production and herd life. Because of differences in production level, the opportunity costs vary considerably among farms. Therefore, PROFOP must be calculated within farms.

Profit equations should represent total profit during the herd life of the cow, so when differences in herd life between cows exist, OPCOST must be considered.

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APPENDIX

Derivation of regression coefficients for three profit equations on production and herd life.

Symbols:

PRTOT = total lifetime profit,
PROFOP = total lifetime profit accounting for opportunity costs (= PRTOT − H × O),
PPD = profit per day of herd life (= PRTOT/H),
H = herd life,
X = first lactation milk production,
$\bar{H}$, $\bar{X}$ = means of H and X,
O = opportunity costs per unit of herd life,
σ^2_H, σ^2_X = phenotypic variance of H and X,
$\sigma_{X,H}$ = phenotypic covariance of X and H, and
$b_{H,PRTOT}$ = regression coefficient of H on PRTOT.

1 Total lifetime profit (PRTOT)

$$\begin{bmatrix} b_{X,PRTOT} \\ b_{H,PRTOT} \end{bmatrix} = \begin{bmatrix} \sigma^2_X & \sigma_{X,H} \\ \sigma_{X,H} & \sigma^2_H \end{bmatrix}^{-1} \begin{bmatrix} \sigma_{X,PRTOT} \\ \sigma_{H,PRTOT} \end{bmatrix}$$

$$= D^{-1} \begin{bmatrix} \sigma^2_H & -\sigma_{X,H} \\ -\sigma_{X,H} & \sigma^2_X \end{bmatrix} \begin{bmatrix} \sigma_{X,PRTOT} \\ \sigma_{H,PRTOT} \end{bmatrix}$$

where $D = \sigma^2_H \sigma^2_X - \sigma_{X,H}\sigma_{X,H}$

$$b_{H,PRTOT} = D^{-1} (\sigma^2_X \sigma_{PRTOT,H} - \sigma_{X,H}\sigma_{PRTOT,X}) \quad [A1]$$

$$b_{X,PRTOT} = D^{-1} (\sigma^2_H \sigma_{PRTOT,X} - \sigma_{X,H}\sigma_{PRTOT,H}) \quad [A2]$$

2 Profit per day (PPD)

$$\begin{bmatrix} b_{X,PPD} \\ b_{H,PPD} \end{bmatrix} = \begin{bmatrix} \sigma^2_X & \sigma_{X,H} \\ \sigma_{X,H} & \sigma^2_H \end{bmatrix}^{-1} \begin{bmatrix} \sigma_{X,PPD} \\ \sigma_{H,PRTOT} \end{bmatrix}$$

PPD = PRTOT/H

Using the result of Pearson (10), we can show that

$$\sigma_{X,PPD} = \sigma_{X,(PRTOT/H)} = C\ \{\sigma_{X,PRTOT}/\overline{PRTOT} - \sigma_{X,H}/\overline{H}\}$$

$$\sigma_{X,PPD} = \sigma_{H,(PRTOT/H)} = C\ \{\sigma_{H,PRTOT}/\overline{PRTOT} - \sigma^2_H/\overline{H}\} \quad [A3]$$

$$b_{X,PPD} = D^{-1}C\{\sigma^2_H\ (\sigma_{X,PRTOT}/\overline{PRTOT} - \sigma_{X,H}/\overline{H}) - \sigma_{X,H}(\sigma_{H,PRTOT}/\overline{PRTOT} - \sigma^2_H/\overline{H})\}$$

$$b_{H,PPD} = D^{-1}C\{-\sigma_{X,H}(\sigma_{X,PRTOT}/\overline{PRTOT} - \sigma_{X,H}/\overline{H}) + \sigma^2_X(\sigma_{H,PRTOT}/\overline{PRTOT} - \sigma^2_H/\overline{H})\} \quad [A4]$$

where $C = \overline{PRTOT}/\overline{H}$.

3 Lifetime profit accounting for opportunity costs (PROFOP)

$$\begin{bmatrix} b_{X,PROFOP} \\ b_{H,PROFOP} \end{bmatrix} = \begin{bmatrix} \sigma^2_X & \sigma_{X,H} \\ \sigma_{X,H} & \sigma^2_H \end{bmatrix}^{-1} \begin{bmatrix} \sigma_{X,PROFOP} \\ \sigma_{H,PROFOP} \end{bmatrix}$$

Using PROFOP = PRTOT − H × O, we can show that

$$\sigma_{X,PROFOP} = \sigma_{\{X,(PRTOT\ -\ H\ \times\ O)\}} = \sigma_{X,PRTOT} - O\sigma_{X,H}$$

$$\sigma_{H,PROFOP} = \sigma_{\{H,(PRTOT\ -\ H\ \times\ O)\}} = \sigma_{H,PRTOT} - O\sigma^2_H$$

$$b_{H,PROFOP} = D^{-1}\ \{\sigma^2_X\sigma_{H,PRTOT} - \sigma_{X,H}\sigma_{X,PRTOT} - O(\sigma^2_X\sigma^2_H - \sigma_{X,H}\sigma_{X,H})\}$$

$$= D^{-1}\ \{\sigma^2_X\sigma_{H,PRTOT} - \sigma_{X,H}\sigma_{X,PRTOT}\} - O \quad [A5]$$

$$b_{X,PROFOP} = D^{-1}\ \{\sigma^2_H\sigma_{X,PRTOT} - \sigma_{X,H}\sigma_{H,PRTOT} + O(\sigma^2_H\sigma_{X,H} - \sigma^2_H\sigma_{X,H})\}$$

$$= D^{-1}\ \{\sigma^2_H\sigma_{X,PRTOT} - \sigma_{X,H}\sigma_{H,PRTOT}\} \quad [A6]$$

From equations [2] and [6], we can conclude that the regression coefficient for production (X) is the same for profit equations PRTOT and PROFOP. The difference in the regression coefficient for herd life (H) is equal to the opportunity cost per unit of herd life (O), as can be seen from Equations [A1] and [A5]. No relation appears to exist between the regression coefficients for PPD and for PRTOT or PROFOP.